

1. This question asks you to use a standard New Keynesian model to interpret changes in inflation persistence in the U.S. Parts (a)-(g) are background algebra, while parts (h) and (i) ask for interpretation. Throughout, you can assume the result in all previous parts of the question, even if you have not been able to derive the result. Total points for this question are 100. The value of each part of the question is given in the left margin, in parentheses.

Let π_t denote inflation, \tilde{y}_t denote the output gap (difference between output and flexible price output), i_t the nominal interest rate. Consider the following New Keynesian system

$$(1) \quad \tilde{y}_t = E_t \tilde{y}_{t+1} - (i_t - E_t \pi_{t+1}) + u_t,$$

$$(2) \quad \pi_t - \pi_{t-1} = \beta(E_t \pi_{t+1} - \pi_t) + \kappa \tilde{y}_t,$$

$$(3) \quad i_t = \delta_\pi \pi_t + \delta_y \tilde{y}_t,$$

where u_t is zero mean and i.i.d, $0 < \beta < 1$, $\kappa > 0$, $\delta_\pi > 1$, $\delta_y \geq 0$.

(5) a. Briefly interpret (1).

(10) b.(i) Briefly interpret

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t,$$

which is a simplified form of (2).

(ii) Now build on your answer to b(i) to briefly interpret (2).

(5) c. Briefly interpret (3).

With straightforward though tedious algebra, equations (1)-(3) may be reduced to a single (dynamic) equation in π_t :

$$(4) \quad \beta E_t \pi_{t+2} - [(1+\beta) + (1+\delta_y)\beta + \kappa] E_t \pi_{t+1} + [\kappa \delta_\pi + 1 + (1+\delta_y)(1+\beta)] \pi_t - (1+\delta_y) \pi_{t-1} = \kappa u_t.$$

(5) d. Say in words how (4) can be derived. That is, you do *not* need to derive (4) formally, just describe the strategy to derive it; but if you prefer to work through the algebra, that is acceptable.

It may be shown that there is a unique stationary solution to (4), which is of the form

$$(5) \quad \pi_t = a \pi_{t-1} + b u_t, \text{ where } a \text{ and } b \text{ are functions of model parameters, and } |a| < 1.$$

(20) e.(i) Show that a satisfies the following polynomial, whose the coefficients match those on the left hand side of (4):

$$(6) \quad \beta a^3 - [(1+\beta) + (1+\delta_y)\beta + \kappa] a^2 + [\kappa \delta_\pi + 1 + (1+\delta_y)(1+\beta)] a - (1+\delta_y) = 0.$$

(ii). Solve explicitly for b (defined in equation (5)) in terms of a and model parameters.

(5) f. Explain why as $\delta_\pi \rightarrow \infty$, with all other parameters held fixed, then $a \rightarrow 0$. (Hint: divide (6) by δ_π , and think about what happens as $\delta_\pi \rightarrow \infty$.)

(10) g. (i) Explain why as $\delta_y \rightarrow \infty$, with all other parameters held fixed, then a tends to satisfy

$$(7) \quad \beta a^2 - (1+\beta)a + 1 = 0.$$

(Hint: divide (6) by δ_y , and think about what happens as $\delta_y \rightarrow \infty$.)

(ii) Taking g(i) as given, explain why $a \approx 1$ when δ_y is very large.

Here are first order autocorrelations for U.S. inflation (GDP deflator, quarterly data) for two different time periods:

1960:1-1982:4	1992:1-2008:4
0.88	0.24

(20) h. In terms of the model developed above, explain how changes in monetary policy could lead to the fall in the first order autocorrelation.

(20) i. Does this model's explanation make sense, in terms of your understanding of U.S. monetary policy? Discuss more generally why inflation persistence might have fallen in the later sample.

2. All parts are equally weighted.

- A. At any date t , a consumer has x_t units of a non-storable good. He can consume $c_t \in [0, x_t]$ of this stock, and plant the remaining $x_t - c_t$ units. He wants to maximize:

$$E \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}$$

where $0 < \gamma < 1$ and $0 < \beta < 1$. Goods planted at date t yield $A_t(x_t - c_t)$ as of the beginning of period $t + 1$, where A_t is a sequence of i.i.d. random variables that take the values of $0 < A_h < 1/\beta$ with probability π and $A_l \in (0, A_h)$ with probability $1 - \pi$.

- i. Formulate the consumer's utility maximization problem in the space of shock-contingent consumption sequences. Exactly what is this space? Exactly what does the expectations operator $E(\cdot)$ mean here? Be explicit.
 - ii. State the Bellman equation for this problem. It is easiest to have the consumer choose savings $s_t = x_t - c_t$. Argue that the relevant state variable for the problem is the cum-return wealth $A_{t-1}s_{t-1}$. Prove (or cite the relevant conditions and results) that the optimal value function is continuous, increasing, and concave in this state.
 - iii. Solve the Bellman equation and obtain the corresponding optimal policy function. (Hint: guess that the optimal function consists of saving a constant fraction of wealth.)
- B. Consider the following economy composed of two agents, A and B , who have deterministically alternating endowments which sum to one. That is, in even periods $t = 0, 2, 4, \dots$ we have $e_t^A = e$ and $e_t^B = 1 - e$ and in odd periods $t = 1, 3, 5, \dots$ we have $e_t^A = 1 - e$ and $e_t^B = e$ where $1/2 < e < 1$. Both agents are risk neutral, but have different discount factors. In particular, the preferences of agents A and B are respectively:

$$U^A = \sum_{t=0}^{\infty} \beta^t c_t^A, \quad U^B = \sum_{t=0}^{\infty} \gamma^t c_t^B$$

where $0 < \gamma < \beta < 1$.

- i. First analyze Pareto optimal allocations. Consider the case of a planner who solves:

$$v(\theta) = \sup_{\{c_t\}} [\theta U^A + (1 - \theta) U^B]$$

subject to the feasibility constraint. Here $0 < \theta < 1$. Find $v(\theta)$ and the consumption sequences that attain $v(\theta)$.

- ii. Now consider a decentralization in which there is a market at date zero for consumption claims, with p_t being the price of consumption at date t . For any θ , construct an equilibrium price sequence so that the equilibrium allocation is the Pareto optimal one. Is there a unique price sequence which decentralizes the optimum?

3. Assume that

output y_t is determined by

$$(1) \quad y_t = \alpha(L)y_{t-1} + \beta(L)m_{t-1} + \varepsilon_t$$

where ε_t is white noise, the money supply m_t is determined by

$$(2) \quad m_t = \eta(L)y_t.$$

and the policymaker wishes to minimize

$$(3) \quad \int_{-\pi}^{\pi} \theta(\omega) f_y(\omega) d\omega$$

where $f_y(\omega)$ is the spectral density of y and $\theta(\omega)$ is a nonnegative weighting function

i. (20) Under what circumstances does the optimal choices of coefficients in (2) convert output to white noise?

ii. (30). In this model, is it possible to test the proposition that the feedback rule is optimally set? Assuming that the feedback rule is optimal, can the policymaker's preferences be recovered?

iii. (30). Are economies characterized by (1) and economies characterized by

$$(4) \quad y_t = \alpha(L)y_{t-1} + \beta(L)(m_{t-1} - E_{t-1}m_{t-1}) + \varepsilon_t$$

observationally equivalent given the feedback rule (2)? Can observational equivalence be affected if the assumption about the money process (2) relaxed?

iv. (20) Assume the special case for the output process

$$(5) \quad y_t = \alpha y_{t-1} + m_{t-1} + \varepsilon_t$$

Suppose that α is known to lie in the interval $[k_1, k_2]$. Suppose that the policymaker wishes to minimize the variance of output. Compare two policies: a passive policy where $\eta = 0$ and an active policy where $\eta = -1/2$. Characterize the minimax and minimax regret policy choices as functions of $[k_1, k_2]$. Can the choices differ?