Microeconomics Prelim — Question 1

Consider an economy with two periods, $t = 0, 1$, three equally probable states of nature, $s = 1, 2, 3$, a single consumption good, and two consumers, $i = 1, 2$. Consumption happens in the second period. State contingent endowments are given by

$$e_1(s) = \begin{cases} 1 & s = 1 \\ 2 & s = 2 \\ 3 & s = 3, \end{cases} \quad \text{and} \quad e_2(s) = \begin{cases} 3 & s = 1 \\ 2 & s = 2 \\ 1 & s = 3. \end{cases}$$

Consumers maximize expected utility with von Neumann-Morgenstern utility function $u_i(c) = \ln c$, $i = 1, 2$, where c represents consumption of the good.

(a) (1 point) Suppose there is a complete set of markets for contingent commodities in period 0. Give a formal definition of the competitive equilibrium for this economy.

(b) (4 points) Find a competitive equilibrium. Justify your answer.

Now suppose that there are no markets for contingent commodities in period 0. Instead, consumers can trade two securities $k = 1, 2$ in period 0. Security k is a title to receive $a_k(s)$ units of the good in state s. The two securities are defined as follows:

$$a_1(s) = \begin{cases} 1 & s = 1 \\ 1 & s = 2 \\ 0 & s = 3, \end{cases} \quad \text{and} \quad a_2(s) = \begin{cases} 0 & s = 1 \\ 1 & s = 2 \\ 1 & s = 3. \end{cases}$$

There is zero net supply of securities and consumers can sell the securities short.

(c) (2 points) Provide the definition of a Radner equilibrium for this economy.

(d) (3 points) Calculate the equilibrium allocation and the equilibrium security prices. Justify your answer.

A monopolist has installed capacity y_t in year t. Each year is divided into two seasons, #1 and #2. Let $\vec{q}_t = (q_{1t}, q_{2t})$ denote the vector of quantities produced by the monopolist in the two seasons in year t. Let $\pi(\vec{q}_t) := R_1(q_{1t}) + R_2(q_{2t}) - c(q_{1t} + q_{2t})$ where ":=" denotes "equal by definition" and $R_1(q_1), R_2(q_2)$ are revenue functions. Here "c" denotes constant marginal cost so that $cq_{1t} + cq_{2t} = c(q_{1t} + q_{2t})$. The monopolist in year t solves the problem

(1) $J(y_t) := \max_{(q_{1t}, q_{2t})}\{R_1(q_{1t}) + R_2(q_{2t}) - c(q_{1t} + q_{2t})\}$, subject to $q_{1t} \le y_t, q_{2t} \le y_t$

i. (1 point) Suppose $R_1(q_1), R_2(q_2)$ are concave revenue functions. Show that $J(y_t)$ is concave in y_t. Hint: Don't use calculus.

ii. (1 point) Suppose the revenue functions are of the form $R_i(q_i) = A_i q_i^{1-\alpha_i}$, $i = 1,2$. What kind of demand generates this form of revenue function? What is its elasticity? What restrictions must you put on this elasticity so that each revenue function is concave increasing?

iii. (2 points) Consider the special case of the revenue functions in part 2, i.e. assume $\alpha_1 = \alpha_2 = \alpha$, $0 < \alpha < 1$ and $A_1 = A_2 + B > A_2 > 0$, so that season #1 is the high demand season and season #2 is the low demand season. Give an analysis of the maximum profit function $J(y)$ in Equation (1) above as a function of y. Sketch a graph of $J(y)$ as a function of y and explain the economics you see in that graph. Is $J(y)$ differentiable at all y's? If not why not? Explain. If you increase y gradually from a very small number what happens to the prices that the monopolist charges for the "high season" demand and the "low season" demand for the case where B is large?

iv. Continue (for your ease even though it is not necessary) to use the special parametric revenue functions of part 3 above. Consider the dynamic problem of capacity accumulation below,

(2) $\max\{\sum_{t=1}^{\infty} \beta^{t-1}(J(y_t) - c_1 x_t - c_2 x_t^2 / 2)\}$, $s.t.\ y_{t+1} = y_t(1-\eta) + x_t, t = 1,2,\ldots; y_1 = y_{10}$.

where $y_{10} > 0$ is very small and positive. Here $0 < \eta < 1$ is a depreciation factor, $0 < \beta < 1$ is a discount factor, and $c_1 x_t + (c_2/2)x_t^2$, $c_1 \ge 0, c_2 \ge 0$ is a cost function for adding capacity, $x_t \ge 0$.

iv.1 (2 points) If $c_1 = c_2 = 0$ solve for the optimal capacity sequence $\{\hat{y}_t\}$ in Problem (2) above. Hint: Use economic common sense to make a conjecture about how the optimal solution of problem (2) behaves. It may help to clear your mind to do the case $A_2 = 0$ first, but this is not necessary.

iv.2 (2 points) Let $c_2 = 0$. What can you say about the optimal solution to problem (2) above? Give a reasonably complete analysis of the time path of optimal capacity over time.

iv.3. (2 points) Now consider the general problem (2) above. Write out the First Order Necessary Conditions (FONC) for an optimal solution. Now make some intelligent conjectures about the time series behavior for the optimal capacity you might expect for the general problem. You are NOT expected to completely solve the general problem.

Consider the following two-player normal form game, G.

		2: a	2: b
1	A	-2, -2	5, 1
	B	1, 5	2, 2

i. Find all Nash equilibria of G.

Let Γ_0 be the following extensive form game. First, player 1 chooses from $\{A, B\}$. Then, after observing player 1's choice, player 2 chooses from $\{a, b\}$. The payoffs to each pure strategy profile are those from the table above.

ii. Compute all subgame perfect equilibria of Γ_0.

Let Γ_1 be the following extensive form game, in which player 2 has the ability to make binding commitments: To start the game, player 2 announces message m_a or m_b. After this public announcement, Γ_0 is played. The payoffs are as in Γ_0, except for the following change: if player 2's action in the last stage does not correspond to his message in the first stage, he pays a penalty of 4.

iii. Compute all subgame perfect equilibria of Γ_1.

Let $\Gamma_2(\alpha)$ be the following extensive form game: To begin, Nature decides whether player 2 is of the "normal" type, θ_n, who cannot make binding commitments, or of type θ_c, who can. The probability that Nature selects type θ_n is $\alpha \in [0, 1]$. Player 2 learns her own type, but this type is not observed by player 1.

After player 2 learns her type, play proceeds as in Γ_1: player 2 announces m_a or m_b, player 1 chooses A or B, and finally player 2 chooses a or b, with each of these choices being publicly observed. If player 2 is of type θ_n, then payoffs are as in Γ_0: in particular, they do not depend on player 2's announcement. If instead player 2 is of type θ_c, then payoffs are as in Γ_1.

iv. Suppose that α is small but positive. Compute all sequential equilibria of $\Gamma_2(\alpha)$.

v. Give an intuitive explanation of any differences between your answers to parts (iii) and (iv).

A trading game.

Consider the following game between a buyer and a seller. The buyer has a valuation v distributed uniformly on [0,1] and seller has a cost c distributed uniformly on [0,1]. The valuation v and the cost c are private information to buyer and seller, respectively. If trade takes place at price p, then the utility function of buyer and seller is v-p and p-c; if there is no trade, both buyer and seller receive the utility of 0

(a) (2pt.) Describe the *ex post* efficient trading rule. Calculate the expected net surplus of the ex post efficient trading rule.

(b) (2pt.) Consider the following mechanism. Buyer and seller offer a bid and a ask price, p_B and p_S, respectively. If $p_B \geq p_S$, then trade takes place at the price $p=0.5*(p_B+p_S)$. Write down the game as a Bayesian game and define the Bayesian Nash equilibrium for this game.

(c) (2pt.) Find an equilibrium in linear strategies for the buyer and the seller with the pricing rule given in part (b). Calculate the coefficient of the linear trading strategy equilibrium.

(d) (4pt.) Consider next the following fixed price strategy by each player

$$p_B(v) = \begin{cases} 0 & \text{if } v < p \\ p & \text{if } v \geq p \end{cases}$$

and

$$p_S(c) = \begin{cases} p & \text{if } p \geq c \\ 1 & \text{if } p < c. \end{cases}$$

for some p in [0,1]. Do these strategies constitute a Bayesian Nash equilibrium for some p? For what values of p do these strategies constitute a Bayesian Nash equilibrium? Compare the equilibria you found (if any) with the allocation in the the *ex post* efficient trading rule?