UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MICROECONOMIC THEORY

August 4, 1998

9:00 a.m. – 1:00 p.m.

INSTRUCTIONS

Answer four of the following six questions. On the top of each page containing your answers write (1) your **assigned number**, (2) the number of the question you are answering, and (3) the position of the page in the sequence of pages used to answer the question.

Example: "#011, Question 1, page 2 of 3".

Do not answer more than one question on the same page. **DO NOT** write your name anywhere on your answer sheets! After the examination, the question sheets and answer sheets will be collected. Please do not write on the question sheets.

Read the questions carefully. Questions will not be explained. If a question seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that are to the point of the question and that demonstrate your command of the relevant economic theory.

1. Consider a one-consumer, one-firm economy with two goods. The consumption set of the consumer is $X^i = \mathcal{R}^2_+$. Her endowment is $(\omega, 0)$, where ω is a positive integer, and her utility function is strictly increasing, strictly quasiconcave, and continuous. The firm produces good 2 from good 1 according to the following technology: for each integer k, in order to increase output of good 2 from k to $k+z$ for any $z \leq 1$, a cost of one unit of good 1 must be paid. (Thus, it takes $k+1$ units of good 1 to produce $k+z$ units of good 2, for all $z \in (0,1]$.) There is free disposal of output.

(a) Draw a picture of the production set for this economy.

(b) Does the first welfare theorem hold in this economy? If so, prove it. If not, provide a carefully labeled counterexample.

(c) Give two examples showing that although a Walrasian equilibrium may be Pareto optimal, the second welfare theorem does not actually hold.

(d) Does a Walrasian equilibrium necessarily exist for this economy? Why or why not?

(e) If this economy had many firms and many consumers, discuss how your answer to part (d) might change.

2. Consider the following mechanism for supplying a public good. Suppose the government announces a proposal to build a public good at a total cost of $\$Y$. It will ask individuals for voluntary donations g_i. Let $G = \sum_i^n g_i$ be the sum of the donations across all n individuals. The government's rule is the following: If $G \geq Y$, then the government will use Y of the money to build the public good, and any surplus $G - Y$ will be burned. However, if $G < Y$ then there will be no public goods provided and all donors will get their donations returned to them. Assume that individuals are endowed with money m_i, and $m_i < Y$ for all i.

(a) Show that if there is a Pareto improving way to provide the public good at the level Y then there will exist a Nash equilibrium such that $G = Y$ and the public good is provided. Will the Nash equilibrium be unique?

(b) Show that regardless of whether there exists a Pareto improving way to provide Y there will always exist at least one equilibrium in which the public good is <u>not</u> provided.

(c) Show that if the public good is provided, then it must be that it is Pareto improving to do so.

(d) Could the public good ever be over-provided with this mechanism? That is, could there be a $Y' < Y$ that is Pareto superior to Y?

(e) Now suppose that the government does not know what levels of Y are Pareto efficient. Design a mechanism that would allow the government to run a series of games, the result of which can eventually lead to a Pareto efficient allocation.

3. A seller has one unit of a good that a buyer values at $v > 0$. The seller derives no value from consuming the good. There is an investment opportunity that can increase the value of consumption for the buyer by α at a cost of k. Assume that $\alpha > k > 0$. Depending on the nature of the investment, the investment opportunity can be available either to the buyer (e.g., contracting to procure a complementary good) or to the seller (e.g., investing in increasing the quality of the good). The first two questions below concern the former scenario while the latter two questions assume the latter. In all cases, the investment decision is made first and then the seller makes a take-it-or-leave-it offer, whose acceptance by the buyer results in a trade at the offered price and whose rejection results in no transfer of the good or money. Both parties are assumed to be risk neutral.

(a) Suppose that the buyer makes the investment decision and that the investment decision is observable to the seller. Find all subgame perfect equilibria. (If it is unique, provide an argument proving it.)

(b) Again the buyer makes the investment decision, but now the seller does not observe the investment activity when the latter makes the offer. Find all subgame perfect equilibria. (If it is unique, provide an argument proving it.)

(c) Suppose now that the seller makes the investment and that the investment decision is observable to the buyer. Find all subgame perfect equilibria. (If it is unique, provide an argument proving it.)

(d) Again the seller makes the investment, but now the investment decision is unobservable to the buyer when the seller makes the offer. Find all subgame perfect equilibria. (If it is unique, provide an argument proving it.)

4. You have a decision to make. When should you buy a computer? Let the value of the computer to you at time $t > 0$, where date 0 is today, be given by $V(t) = \exp[At]V(0)$, where $V(0) > 0$ and A is nonnegative. When $A > 0$ the value rises due to technical improvements in hardware and available software. The purchase cost is given by $C(t) = \exp[-Bt]C(0)$, $C(0) > 0$, and B is nonnegative. When $B > 0$, purchase cost is falling due to technical change in production. Let

$$W(T) = \int_T^\infty \exp[-Rt]V(t)\,dt. \tag{1}$$

You wish to maximize

$$J(T) = W(T) - \exp[-RT]C(T). \tag{2}$$

(a) Write out the formula for $W(T)$ and locate sufficient conditions on (A, R) on this problem for $W(T)$ to be finite for all finite positive T.

(b) Now write out the first order necessary conditions for an interior optimum T, call it T^*. Check the second order necessary conditions to make sure that it is a maximum you found and not a minimum. Find T^*. Find sufficient conditions for optimum T to be zero (i.e. purchase your computer now). Find sufficient conditions for the optimal T to be positive and finite. Find sufficient conditions for the optimal T to be plus infinity, i.e. never purchase a computer.

(c) Increase B. What happens to the optimum solution T^* when you increase B? Does it increase or decrease? Increase A. What happens to T^*?

(d) Now interpret this problem as finding the optimum time T to build a facility that generates value stream $\{V(t), \forall t \in [T, \infty]\}$ and costs $C(T)$ if built at date T. Suppose two firms compete to build the facility. One type of competition leads to T_c that solves equation (3) below,

$$\int_T^\infty \exp[-Rt]V(t)\,dt = \exp[-RT]C(T). \tag{3}$$

Find T_c that solves (3) and compare it with T^* that solves (2). Find a sufficient condition to obtain a positive solution T_c to (3). Under this sufficient condition which is larger, T^* or T_c?

6. Consider a job market signaling model in which workers are of type $t = 1$ or $t = 2$. The share of workers with type $t = 1$ is $q \in (0, 1)$. Type t workers have productivity $y(e, t) = t$, where e is the education level of the worker. The cost of education to a worker of type t is $c(e, t) = \frac{e}{t}$. Each worker first chooses a level of education to invest in. His wage w is then set in a competitive labor market in which firms cannot see a worker's type, but can see his level of education. A worker with type t who chooses education level e and receives a wage w obtains utility $w - c(e, t)$.

(a) Derive the set of outcomes (e, w) for all perfect Bayesian equilibria in which workers pool.

(b) Derive the set of PBE pooling outcomes which are supported by equilibria in which firms, when possible, do not believe a worker has played a strictly dominated strategy. Will such an equilibrium always exist?

(c) Derive the set of separating perfect Bayesian equilibrium outcomes. Which of these outcomes (if any) can be supported by equilibria in which firms, when possible, do not believe a worker has played a strategy which is equilibrium dominated? (i.e., which equilibrium outcomes survive the refinement known as the "intuitive criterion?") Which separating PBE outcome is Pareto dominant?

5. A factor of production is called *inferior* if the conditional demand for that factor falls as output is increased while factor prices are held constant.

(a) Draw an isoquant map showing a technology with an inferior factor of production.

(b) Is it possible that an increase in the price of some factor of production might cause a profit maximizing firm to **increase** output? Explain.

(c) Analyze the following argument: "In consumer theory it can happen that an increase in the price of some good leads to an increase in the quantity demanded (if the good is inferior and the income effect of the price increase is stronger than the substitution effect). Similarly it can happen that an increase in the price of a factor of production leads to an increase in the quantity of that factor demanded by a profit-maximizing firm. This can happen if the higher factor price leads to a reduction in output, and if the factor is inferior, so that the reduction in output leads to an increase in the use of this factor."