

**UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS**

MICROECONOMIC THEORY

August 5, 1997

9:00 - 1:00 pm

Instructions

On the top of each yellow sheet, write your **ASSIGNED NUMBER**, date, and name of exam and number of each question. DO NOT write your name on the yellow pads. After the examination, the questions sheets and yellow pads will be collected. Do not write on the question sheets.

Read the questions carefully. Questions will not be explained. If a question seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well-organized and legible answers that are to the point of the question and demonstrate your command of the relevant economic theory.

Please return unused portion of yellow tablets.

ANSWER 4 OUT OF THE FOLLOWING 6 QUESTIONS

QUESTION 1. Let $\{x_1(P, m), x_2(P, m), \dots, x_n(P, m)\}$ be demand functions for n goods obtained from the classical consumer's problem.

$$\max_X u(X)$$

subject to

$$P \cdot X = m,$$

where $P = (p_1, p_2, \dots, p_n)$, $X = (x_1, x_2, \dots, x_n)$, and $m =$ income.

a. Suppose that for some good i , we have, for all (P, m) ,

$$x_i(P, m) = h_i(P)g_i(m),$$

for some pair of functions $h_i(\cdot)$, $g_i(\cdot)$.

Show that $g_i(m)$ must be of the form:

$$g_i(m) = m^{e_i},$$

for some constant e_i . That is, g_i is income raised to the e_i -th power so that income elasticity is given by the constant e_i .

b. Suppose all of the demand functions are of the form $x_k(P, m) = h_k(P)g_k(m)$, $k = 1, 2, \dots, n$. Show that Marshall Cross Partial

$$\partial x_i / \partial p_j = \partial x_j / \partial p_i,$$

for all pairs i, j with equal income elasticities. You may use result a. for this part.

QUESTION 2. Consider a duopoly market in which firms 1 and 2 produce homogeneous products. If the firms choose quantities q_1 and q_2 , then the market price is $\max\{1 - q_1 - q_2, 0\}$. There are no costs.

a. Suppose that firm 1 first makes a quantity choice, firm 2 observes this choice, and then firm 2 makes a choice, at which point output is sold and profits realized. Find a Nash equilibrium of this game that is not subgame perfect as well as a subgame perfect equilibrium.

b. Now suppose that firm 2 cannot observe q_1 perfectly, instead observing $q_1 + \epsilon$, where ϵ is a random variable with full support on the real line. Find the pure-strategy equilibria of this game. Be precise about what equilibrium concept you are using.

c. Suppose instead that firm 2 either observes q_1 perfectly or not at all. The probability of observing q_1 is $1/2$. When choosing q_1 , firm 1 does not know whether firm 2 will observe q_1 . What is the equilibrium of this game?

QUESTION 3. Consider a simple economy in which farming is the only occupation. All arable land lies along a linear river, and is divided into 2000 farms, arranged symmetrically on each side of the river. Up-river the weather is not so warm, and the beaches are further away, so each farmer would prefer to be closer to the sea, *ceteris paribus*. There are 105 farmers, each of whom can work just one farm, and each farm yields y pounds of food. Each farmer's utility function is $U(c, x) = \log(c) - x^2$, where c is consumption of food, and x is distance from the coast, in farms ($x = 1, 2, \dots, 1000$). Each farmer owns one share in each farm.

Find a competitive equilibrium for this economy, and draw a diagram summarizing how it works. Is the equilibrium unique?

QUESTION 4. Consider the following bargaining model in which the quality of the good is known only to the seller. Buyer and seller have valuations v and c , respectively. The seller knows her valuation c , but the buyer views it as distributed uniformly on $[0, 1]$. The buyer does not know his own valuation, but knows that it is a function of the seller's valuation; in particular $v = \sqrt{kc}$, where the parameter $k > 0$ is common knowledge.

- a. What is the efficient (in the full information sense) trading outcome?
- b. Suppose the buyer and seller play an ultimatum game in which the buyer makes a take-it-or-leave-it offer to the seller. For each $k > 0$, carefully describe the set of subgame perfect equilibria of this game. Prove that the strategies you identify actually do give subgame perfect equilibria.
- c. For what values of k will a subgame perfect equilibrium of the game in part b yield an efficient outcome?
- d. Suppose instead that the buyer's valuation is given by $c + k$, where $k \in (0, 1)$ is common knowledge. Show whether it is more efficient to have the buyer or seller make the ultimatum offer.

QUESTION 5. Consider a representative individual with preferences

$$U(x_0, x_1, x_2) = x_0 + \alpha \ln(x_1) + \beta \ln(x_2).$$

Suppose individuals are endowed with m units of good x_0 (time, for instance), which can be turned into the other goods at a constant rate. Hence, the technology implies a linear budget constraint $m = x_0 + c_1x_1 + c_2x_2$. (Note, α and β can be quite large.)

a. Find the indirect utility function for this person.

b. Suppose that the government is going to take on a project that requires R in tax revenues. However, the only method available to the government for raising R is unit commodity taxes on goods x_1 and x_2 . Hence the government is constrained by the fact that the endowment of x_0 cannot be taxed directly. As a result, the government wants to choose the level of taxation that both meets its revenue target R , and is optimal in that it leaves the representative individual as well off as possible.

Let t_i be the tax on good i , and $p_i = c_i + t_i$ be the after-tax price of good i . The government must raise $R = t_1x_1^*(p) + t_2x_2^*(p)$, where $x^*(p)$ refers to the Marshallian demands. Hence, the government's problem is to choose the taxes t_1 and t_2 to maximize (indirect) utility, subject to the government revenue requirement.

Write down the government's objective function as a Lagrangian, \mathcal{L} , using λ as the Lagrange multiplier.

c. Show that the taxes that solve this optimization problem imply *uniform commodity taxation*, i.e. $t_1/(c_1 + t_1) = t_2/(c_2 + t_2)$.

d. What economic interpretation would you give to λ in the above problem?

e. Solve for λ and interpret your finding. I.e., is $\lambda > 1$ or is $\lambda < 1$? How does λ compare to $\partial \mathcal{L} / \partial m$? Interpret the difference in an economically meaningful way.

QUESTION 6. A seller has a good whose quality is high or low with equal probabilities. A high quality good is defective with probability $1/3$, and a low quality good is defective with probability $2/3$. A buyer values the good at \$1 when it is not defective and at zero when it is defective. The seller values the good at zero regardless of the quality. Both parties are risk neutral. When selling the good, the seller may simultaneously offer a warranty. A warranty of level $q \in [0, 1]$ obligates the seller to repair the good, when it is defective, at the cost of $2q$ so that the buyer receives the value of q (instead of zero) from that good. Initially, the seller makes a take-it-or-leave-it offer to the buyer of a contract specifying the buyer's payment, t , and the warranty level, q .

a. Suppose that the quality of the good is a common knowledge for both the buyer and the seller. (That is, both parties know the probability of the good being defective at the time of contract offering, but not whether the good is defective or not, which is a random variable at that time.) Characterize the subgame perfect equilibrium outcome of the game. Does the seller of each type provide any warranty in that equilibrium?

b. Suppose now that the quality of the good is private information to the seller, and the buyer knows only the probability that the good is high quality, which is one half. (The seller knows the probability of the good being defective, but not whether the good is defective or not, which is a random variable at the time of contract offering.) Characterize all separating Perfect Bayesian equilibrium outcomes. Show that each equilibrium is supported by a weakly consistent system of beliefs. Argue for the most plausible one, if there exists one. Does the seller of each type offer an warranty in that equilibrium? (Hint: You may draw contracts in the (t, q) plane.)