

University of Wisconsin
Microeconomics Prelim Exam
Monday, August 29, 2011: 9AM - 2PM
Questions and Solutions

- There are four parts to the exam. All four parts have equal weight.
- Answer all questions. No questions are optional.
- **Hand in 12 pages, written on only one side.**
- Write your answers for different parts on different pages. So do not write your answers for questions in different parts on the same page.
- Please place a completed label on the top right corner of each page you hand in. On it, write your assigned number, and the part of the exam you are answering (I,II,III,IV).
- Show your work, briefly justifying your claims. Some solutions might be faster done by drawing a suitable diagram.
- You cannot use notes, books, calculators, electronic devices, or consultation with anyone else except the proctor.
- Please return any unused portions of yellow tablets and question sheets.
- There are five pages on this exam, including this one. Make sure you have all of them.

Part I — The Emerald Isle

Suppose 100 identical consumers have preferences over a basic good, x_0 , and a composite y of different varieties of another good. Each consumer starts with 1000 units of the basic good and has preferences represented by the utility function

$$U(x) = x_0 + y$$

where

$$\sqrt{y} = \sum_{i=1}^n \sqrt{x_i}$$

The price of good x_i is p_i , with $p_0 = 1$.

Each variety x_i is produced by one firm, which requires one unit of capital regardless of how much output is produced, and also uses 1 unit of the basic good for every 10 units of output. The market supply curve of capital is

$$p_K = \frac{1}{8}K$$

where p_K is the price of capital (per unit of the basic good) and K is the quantity supplied.

The number of varieties, n , is determined by free entry: anyone can introduce a new variety, and there will then be a new firm producing that variety. There is an unlimited supply of potential entrants, and they act so as to maximize profits.

1. In equilibrium, how many varieties will be produced?
2. Is the equilibrium efficient?

Solution: This is a hard question, and it was graded accordingly. The utility is

$$U = x_0 + y$$

where we will write

$$y^\rho = \sum_{i=1}^n x_i^\rho$$

with $\rho = 1/2$ in this question, of course, but more generally, $0 < \rho < 1$.

The budget constraint is

$$x_0 + \sum_{i=1}^n p_i x_i = \mu$$

Thus the consumer uses a CES technology to transform the varieties x_i into the composite good y . This technology has constant returns, and the input prices are given. Thus, the consumer effectively buys the composite good at a constant price per unit q given by the CES cost function

$$q^{1-\sigma} = \sum_{i=1}^n p_i^{1-\sigma}$$

where $\sigma = \frac{1}{1-\rho}$.

In a symmetric equilibrium,

$$q^{1-\sigma} = n p_i^{1-\sigma} \quad \Rightarrow \quad \left(\frac{p_i}{q} \right)^{1-\sigma} = \frac{1}{n}$$

for each i .

Since x_0 and y are perfect substitutes, with a marginal rate of substitution of 1, the consumer buys whichever one is cheaper. In the interesting case, $q < p_0$, so the basic good is not consumed. This is the case where the cost of producing the varieties x_i is relatively low (which holds for the parameter values given in the question).

Each producer has a monopoly in the market for x_i , so p_i is chosen so as to equate marginal revenue and marginal cost (c).

By Shephard's Lemma, each individual's demand for x_i is given by

$$x_i = y \frac{\partial q}{\partial p_i}$$

with

$$y = \frac{\mu}{q}$$

from the budget constraint. Differentiating the cost function equation gives

$$(1 - \sigma) q^{-\sigma} \frac{\partial q}{\partial p_i} = (1 - \sigma) p_i^{-\sigma}$$

So

$$\frac{\partial q}{\partial p_i} = \left(\frac{p_i}{q}\right)^{-\sigma} \Rightarrow \frac{\partial \log q}{\partial \log p_i} = \left(\frac{p_i}{q}\right)^{1-\sigma}$$

Thus

$$x_i = \frac{\mu}{q} \left(\frac{p_i}{q}\right)^{-\sigma} \Rightarrow \log x_i = \log \mu - \sigma \log p_i - (1 - \sigma) \log q$$

The elasticity of producer i 's demand curve is found by differentiating this equation with respect to $\log p_i$. In a symmetric equilibrium, this gives

$$\eta = \sigma + (1 - \sigma) \frac{1}{n}$$

where η is the absolute value of the elasticity.

Equating marginal revenue and marginal cost gives

$$p_i \left(1 - \frac{1}{\eta}\right) = c$$

or

$$p_i - c = \frac{p_i}{\eta}$$

The zero profit condition is

$$J(p_i - c)x_i = an$$

where J is the number of consumers. When marginal revenue is set equal to marginal cost, this reduces to

$$Jp_ix_i = a((n - 1)\sigma + 1)$$

where the market supply curve of capital is

$$p_K = aK$$

with $K = 1$ for each active producer. But in a symmetric equilibrium the budget constraint implies

$$p_ix_i = \frac{\mu}{n}$$

Then the zero profit condition reduces to

$$\frac{J\mu}{a} = n((n-1)\sigma + 1)$$

The right side is increasing in n , so a greater number of consumers, or an increase in the endowment of the basic good, must increase the number of firms, while a greater fixed cost decreases the number of firms.

The solution (using the quadratic formula) is

$$n = \frac{\sqrt{(\sigma-1)^2 + 4\sigma J\mu/a} + \sigma - 1}{2\sigma}$$

For the parameter values given in the question, the solution is

$$n = 632.7055815$$

This solution was obtained on the assumption that $q < 1$, but it is easy to verify that this inequality holds, because $q < p_i$, and p_i is roughly twice the marginal cost, which is $\frac{1}{10}$.

EFFICIENCY. Each consumer's utility is just

$$y = n^{\frac{1}{\rho}} x_i = n^{\frac{1}{\rho}-1} \mu/p_i$$

If n is given, then the aggregate resource constraint is

$$\left(\frac{a}{J} + cx\right)n = \mu \quad \Rightarrow \quad x = \frac{1}{c} \left(\frac{\mu}{n} - \frac{a}{J}\right)$$

and utility is

$$y = \frac{n^{\frac{1}{\rho}}}{c} \left(\frac{\mu}{n} - \frac{a}{J}\right)$$

For example, if $\rho = \frac{1}{2}$, then utility is

$$y = \frac{1}{c} \left(\mu n - \frac{a}{J} n^2\right)$$

so the optimal number of varieties is

$$n^* = \frac{J\mu}{2a}$$

For the parameter values given in the question, $n^* = 400,000$. So the monopolistic competition equilibrium is not efficient (nor is there any reason to expect that it would be).

REMARK. This question is based on a well-known paper by Dixit and Stiglitz (1977), but the composite good in that paper is defined as

$$y = \sum_{i=1}^n x_i^\rho$$

REFERENCE. Dixit, Avinash K. and Stiglitz, Joseph E. (1977): Monopolistic Competition and Optimum Product Diversity. *American Economic Review* 67(3), 297–308.

Part II — The True North Strong and Free

1. Miss Homo economicus is indifferent between two different price vectors p and q . How does she compare p to $(p + q)/2$?

Solution: The indirect utility function is quasi-convex, in other words, the lower contour sets are convex. (This follows from properties of the maximization, and does not require convex preferences!) Thus, $(p+q)/2$ lies in the lower contour set of Homo economicus, and is less preferred.

2. The price elasticity of demand for a firm with market power is (minus) 2.3. If its price rises 1%, what happens to its revenue?

Solution: When the price rises 1%, the quantity falls about 2.3%, and thus the revenue falls about 1.3%.

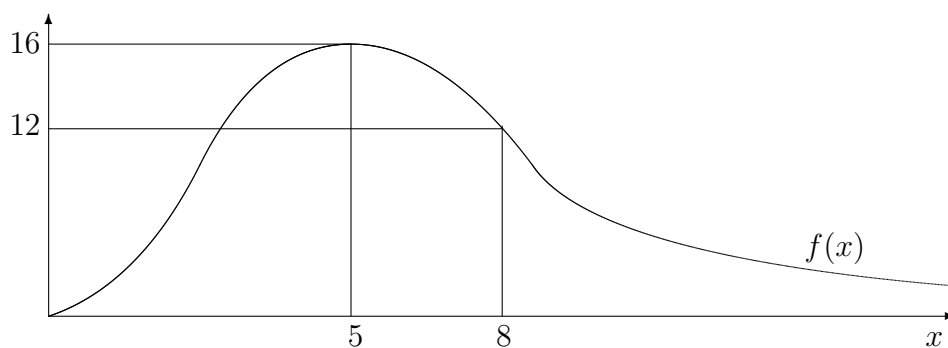
3. Consider a world with a single non-storable consumption good: clams. Everyone is happy as a clam, and lives by the ocean; they harvest clams near the beach next their backdoor, eat clam chowder for lunch, and have a clam-bake at dinner. Everyone is aware that randomly, with chance 10%, THE GREAT CLAM DISEASE (GCD) might wipe out most of the clams. Knowing this, the residents set up a *contingent clams market*, where residents can sell their endowment and buy clams in the event of the GCD for price $p > 0$ in exchange for one clam when no GCD occurs. Is it true that $p = 1/9$? $p > 1/9$? $p < 1/9$?

Solution: Optimality requires that the price ratio equal the ratio of expected marginal utilities. Let's assume state-independent and risk averse utility functions. If $p = 1/9$ (namely, the ratio of probabilities), then individuals would perfectly insure, consuming the same quantity of clams in either state. But the GCD constitutes an aggregate shock, since most clams are wiped out. Thus, prices must lead individuals to consume less clams in this state, and so $p > 1/9$.

4. Assume that my *marginal value of information* is a smooth function $f(x)$ of the quantity x , obeying $f(0) = 0$, with f first strictly increasing, and thereafter strictly positive and decreasing. Assume that my *maximum marginal value* is $f(5) = 16$ and that $f(8) = 12$. Assume also that my *total value* of information is 0 at $x = 0$, and 96 at $x = 8$. Schematically plot the above curve, and on the same diagram, plot the information *demand*, ensuring that it reflects the above facts. Finally,

plot the inverse demand curve (with price on the vertical axis and quantity on the horizontal axis). Make sure your graph addresses the following issues: Does information demand always rise when the price falls? What happens to demand as the price vanishes to zero?

Solution: Plot the marginal value curve f . Since f is hill-shaped, the demand curve is nonstandard. Optimization requires that marginal value be falling (and so after $x = 5$) and obey global optimality, namely, total value weakly exceed total cost, and thus after $x = 8$. (For $x < 8$, total value, or the area below the marginal value curve, is below the cost one would.) In other words, the demand curve coincides with the marginal value curve $f(x)$ for all $x \geq 8$, and thus explodes as the price vanishes. Demand is 0 for all prices over 8. Below I plot the marginal value; the demand curve is the part of the curve right of $x = 8$; and the inverse demand is omitted, but is the reflection in the diagonal of the demand.



Part III — America the Beautiful

1. Players 1 and 2 plan to go skating at a pond. However, skating is only possible if at least one of them shows up early to clear the ice.

Each player i is either a morning person (t_i^m) or night person (t_i^n). Each player knows his own type, and, regardless of his own type, assigns a probability of $\frac{1}{3}$ to the other player being a morning person.

Each player chooses between showing up early (E) to clear the ice or showing up late (L). Payoffs come from two additive factors:

- If either player shows up early, then skating is possible, and each player receives 4. If both show up late, each player receives 0.
- A morning person who shows up early also receives a bonus of 1, whereas a night person who shows up early suffers a penalty -3 .

- (a) Describe this strategic interaction as a Bayesian game, and define each player's set of pure Bayesian strategies. Can the private information in this game be derived from a common prior?

Solution: Identify a player's type with his cost of showing up early: $T_i = \{t_i^m, t_i^n\} \equiv \{-1, 3\}$. Then the game can be described as follows:

common prior		payoffs																						
		player 2																						
		E	L																					
$t_1^m (= -1)$	<table style="border-collapse: collapse; margin: 0 auto;"> <tr> <td style="border: 1px solid black; padding: 5px;">$t_2^m (= -1)$</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">1/9</td> <td style="border: 1px solid black; padding: 5px;">$t_2^n (= 3)$</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">2/9</td> </tr> <tr> <td style="border: 1px solid black; padding: 5px;">$t_1^n (= 3)$</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">2/9</td> <td style="border: 1px solid black; padding: 5px;">$t_2^m (= -1)$</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">4/9</td> </tr> <tr> <td style="border: 1px solid black; padding: 5px;"></td> <td style="border: 1px solid black; padding: 5px; text-align: center;">1/3</td> <td style="border: 1px solid black; padding: 5px;"></td> <td style="border: 1px solid black; padding: 5px; text-align: center;">2/3</td> </tr> </table>	$t_2^m (= -1)$	1/9	$t_2^n (= 3)$	2/9	$t_1^n (= 3)$	2/9	$t_2^m (= -1)$	4/9		1/3		2/3	1/3	1	<table style="border-collapse: collapse; margin: 0 auto;"> <tr> <td style="border: 1px solid black; padding: 5px;">E</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">4 - t_1, 4 - t_2</td> <td style="border: 1px solid black; padding: 5px;">L</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">4 - t_1, 4</td> </tr> <tr> <td style="border: 1px solid black; padding: 5px;">L</td> <td style="border: 1px solid black; padding: 5px; text-align: center;">4, 4 - t_2</td> <td style="border: 1px solid black; padding: 5px;"></td> <td style="border: 1px solid black; padding: 5px; text-align: center;">0, 0</td> </tr> </table>	E	4 - t_1 , 4 - t_2	L	4 - t_1 , 4	L	4, 4 - t_2		0, 0
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L	4, 4 - t_2		0, 0																					

The above representation describes beliefs at the *ex ante* stage using a common prior; one could also start from beliefs at the *interim* stage.

A pure Bayesian strategy for player i is a map $s_i : \{t_i^m, t_i^n\} \rightarrow \{E, L\}$.

- (b) Find all of the Nash equilibria of the game you defined, and then comment on their efficiency properties.

Solution: For the morning people, E is strictly dominant. Therefore, $s_1(t_1^m) = s_2(t_2^m) = E$ in any Nash equilibrium.

Now consider the incentives of a night person:

$$U_1(E, s_2|t_1^n) = 4 - t_1^n = 4 - 3 = 1$$

$$\begin{aligned} U_1(L, s_2|t_1^n) &= p_1(t_2^n|t_1^n) u_1((L, s_2(t_2^n)), (t_1^n, t_2^n)) + p_1(t_2^m|t_1^n) u_1\left((L, \overbrace{s_2(t_2^m)}^E), (t_1^m, t_2^n)\right) \\ &= \begin{cases} \frac{2}{3} \cdot 4 + \frac{1}{3} \cdot 4 & \text{if } s_2(t_2^n) = E \\ \frac{2}{3} \cdot 0 + \frac{1}{3} \cdot 4 & \text{if } s_2(t_2^n) = L \end{cases} \\ &\geq \frac{4}{3} > 1 \end{aligned}$$

We conclude that in any Nash equilibrium, $s_1(t_1^n) = s_2(t_1^n) = L$.

Thus, in the unique Nash equilibrium, the negative cost types work, and the positive cost types don't. So with probability $\frac{4}{9}$, neither player works and no benefit is obtained, despite the fact that all types would work if they were alone. This is clearly inefficient.

2. Let G be a two-player normal form game. Create a new normal form game \hat{G} by sequentially adding strictly dominated strategies to G for one or both players. Let G^∞ and \hat{G}^∞ be the respective infinitely repeated version of G and \hat{G} with some discount factor $\delta \in (0, 1)$.

- (a) Is it possible that G^∞ and \hat{G}^∞ have different sets of subgame perfect equilibrium payoffs? Provide an example in which this happens, or explain why it cannot happen.

Solution: This is possible. For an example, let \hat{G} be a Prisoner's Dilemma, and let G be the Prisoner's Dilemma with the strategy "co-operate" removed for both players.

- (b) Answer part (a) again, this time under the additional assumption that G and \hat{G} are each zero sum.

Solution: In this case the payoffs must be the same. In a one-shot game, adding strictly dominated strategies does not change the set of Nash equilibria. And in a repeated zero-sum game (with payoffs normalized in the usual way), the unique subgame perfect equilibrium payoff is the Nash equilibrium payoff in the one-shot game: this can be attained by repeated play of the Nash equilibrium, while under any other feasible payoff vector, one player gets less than his minmax value and so has a profitable deviation.

Part IV — Polonia: Know your value

Consider a signaling model, in which a company wants to hire a worker. The *expected productivity* θ_i is private information of worker i and not observable to the company. The company operates in a competitive labor market, so that *workers must be paid their expected productivity*.

Suppose there is an equal probability mass of three types of workers $\theta_H > \theta_M > \theta_L$. The costs of education is $c(e, \theta_i) = e/\theta_i$, for $i = H, M, L$.

1. Is there an equilibrium where types θ_L and θ_H choose education level e_L , while θ_M chooses education level $e_M \neq e_L$? If there is, please describe it. If not, please explain why. Then, apart from providing an analytical explanation, use the definition of the single-crossing condition to give a brief intuition for your claim of existence or non-existence.

Solution: The problem set-up is standard and so omitted. We just consider education levels, and omit belief determination.

We argue that no such equilibrium with $e_M \neq e_L = e_H$ exists. Suppose otherwise, so that θ_L chooses education level e_L^ , while θ_M chooses education level $e_M^* \neq e_L^*$. The incentive constraints for types M, L are*

$$\begin{aligned} w_M - \frac{e_M^*}{\theta_M} &\geq w_L - \frac{e_L^*}{\theta_M} \\ w_L - \frac{e_L^*}{\theta_L} &\geq w_M - \frac{e_M^*}{\theta_L}. \end{aligned}$$

Together, these IC constraints imply

$$\theta_M (w_M - w_L) \geq e_M^* - e_L^* \geq \theta_L (w_M - w_L).$$

Therefore, $w_M \geq w_L$. But then θ_H prefers (e_M, w_M) to (e_L, w_L) , since

$$\theta_H (w_M - w_L) \geq \theta_M (w_M - w_L) \geq e_M^* - e_L^*$$

implies:

$$w_M - \frac{e_M^*}{\theta_H} > w_L - \frac{e_L^*}{\theta_H}.$$

This contradicts $e_L = e_H$.

Finally, the single-crossing condition argues that that if a lower type is indifferent between signal-action pairs, then a higher type strictly prefers to send the higher signal. This guarantees that higher types send weakly higher signals in equilibrium.

2. Define and describe the set of equilibria where all three types act identically (i.e. where all three types pool).

Solution: The common wage is competitively determined, and so $E(\theta) = \frac{1}{3}\theta_H + \frac{1}{3}\theta_M + \frac{1}{3}\theta_L$. Let e^ be the education level chosen by all worker types in a pooling equilibrium. The pooling equilibrium requires that the low type not deviate*

$$0 \leq e^* \leq (E(\theta) - \theta_L)\theta_L.$$

The posterior beliefs equal the prior in equilibrium, but must be a point mass on type θ_L for all other education levels $e \neq e^$.*