

University of Wisconsin
Microeconomics Prelim Exam
Friday, June 27, 2011: 9AM - 2PM

- There are four parts to the exam. All four parts have equal weight.
- Answer all questions. No questions are optional.
- Hand in 12 pages, written on only one side.
- Write your answers for different parts on different pages. So do not write your answers for questions in different parts on the same page.
- Please place a completed label on the top right corner of each page you hand in. On it, write your assigned number, and the part of the exam you are answering (I,II,III,IV).
- Show your work, briefly justifying your claims. Some solutions might be faster done by drawing a suitable diagram.
- You cannot use notes, books, calculators, electronic devices, or consultation with anyone else except the proctor.
- Please return any unused portions of yellow tablets and question sheets.
- There are six pages on this exam, including this one. Make sure you have all of them.

Part I — Ireland

Consider a pure exchange economy with three consumers (labeled 1, 2 and 3) and three goods (labeled x, y and z). Agent i 's consumption vector is (x_i, y_i, z_i) . Each agent is endowed with only one type of good: $\omega_1 = (0, 0, 3)$, $\omega_2 = (1, 0, 0)$, and $\omega_3 = (0, 2, 0)$, where ω_i is i 's endowment. The consumers' preferences can be represented by utility functions, as follows:

$$\begin{aligned}U_1 &= \log(y_1) + \log(z_1) \\U_2 &= \sqrt{y_2 z_2} \\U_3 &= x_3 y_3 z_3\end{aligned}$$

1. Find a competitive equilibrium. Is it unique?
2. Find the set of Pareto optimal allocations.

Part II — Canada

1. Mary Zap spends all her budget, and consumes no inferior goods (where consumption falls with income). What is an upper bound on the share of her budget allocated to a good with income elasticity 4?
2. Rockafellar (or Rocky, as his friends call him) has strictly convex preferences over leisure and one other composite consumption good. Rocky's only source of income is a flex-hours job, with a constant hourly wage rate, for however many hours he works. If Rocky faces a *regressive* income tax (i.e. the percentage tax rate he pays is absolutely lower at greater incomes), could he ever be indifferent about two levels of leisure? What if he has a *progressive* income tax?
3. Assume that Leia obeys the axioms of the von Neumann Morgenstern Expected Utility Theorem, and has the increasing and continuous (Bernoulli) utility function u . Assume that Luke's utility of any lottery with prizes x_i having chances p_i is $\sum_i p_i u(x_i) + [\sum_i p_i u(x_i)]^3$. How do their attitudes to risk compare?
4. An economy has just three firms f_1, f_2, f_3 and just three employees e_1, e_2, e_3 , and pairwise production as specified below. When a match of firm f_i and employee e_j occurs, the output is split among the pair, yielding profit π_i to firm i and wage w_j to employee j . A firm can only employ one employee and an employee can only work for one firm.

	f_1	f_2	f_3
e_1	10	0	0
e_2	15	10	0
e_3	17	14	10

Find the efficient matching. Then find the tightest possible upper and lower bounds on the profit of firm 2 in a competitive equilibrium.

Part III — U.S.A.

For questions 1 and 2 below, consider the following interaction between a police officer (player 1) and a motorist (player 2). At the start of the interaction, the police officer observes the motorist speeding. The officer chooses between leaving the motorist alone (L), pulling the motorist over to give her a ticket (T), or pulling the motorist over to arrest her (A). If the officer leaves the motorist alone, then the game ends. If not, then the motorist must decide whether to accept her penalty (a) or drive away (d). When the motorist makes this decision, she only knows that the officer has pulled her over, but cannot tell whether the officer intends to ticket her or arrest her. Whenever the motorist drives away, she is equally likely to get caught (C) and to escape (E). Payoffs are as follows:

- If the officer leaves the motorist alone, then both players receive a payoff of 0.
 - If the officer pulls the motorist over to give her a ticket and the motorist accepts the penalty, then the officer gets 3 and the motorist gets -5 .
 - Arresting the motorist means extra paperwork for the police officer. Therefore, if the officer pulls the motorist over to arrest her and the motorist accepts the penalty, then the officer gets a payoff of just 2, while the motorist gets -10 .
 - If the officer pulls the motorist over and the motorist drives away and is caught, then the officer gets a payoff of 5, while the motorist gets -15 .
 - If the officer pulls the motorist over and the motorist drives away and escapes, then the motorist gets 0; the officer gets -10 if he was pulling the motorist over to ticket her, and -11 if he was pulling her over to arrest her (since he will still have to do the extra paperwork).
1. Construct two extensive form representations, Γ and Γ' , of this interaction such that (a) Γ and Γ' differ only in the presentation of player 1's decision, and (b) the sets of subgame perfect equilibrium outcomes of Γ and Γ' are different. (In doing so, you should compute the sets of subgame perfect equilibria for the two games.)

2. Compute the sequential equilibria of Γ and Γ' . Are the sets of sequential equilibrium outcomes of the two games the same?
3. Consider the normal form game G below. Assume that $a_i - b_i - c_i + d_i \neq 0$ for $i \in \{1, 2\}$, and that G has a unique completely mixed equilibrium, σ . (The game may have additional equilibria that involve pure strategies.)

		2	
		L	R
1	T	a_1, a_2	b_1, b_2
	B	c_1, c_2	d_1, d_2

Suppose we increase payoff entry a_1 in such a way that the sign of $a_1 - c_1$ does not change. Call the resulting game G' . Explain why G' must also have a unique completely mixed equilibrium. Denote this equilibrium by σ' . Specify which components of σ' differ from those of σ . For those that differ, specify the directions of the changes in values as functions of the game's payoff parameters.

Part IV — Poland

Consider the following versions of a signaling model, in which a company wants to hire a worker. Productivity is worker private information and is not observable to the company. The company maximizes expected profits. Worker reservation wage is equal to 0. The labor market is competitive.

1. Suppose that a worker has productivity $\theta \in \{\theta_L, \theta_H\}$, $\theta_H > \theta_L$, and $\Pr(\theta = \theta_H) = \mu$. Worker chooses an education level $e \in [0, \infty)$ and is paid $w(e)$. A worker's utility is given by $U(w, e, \theta) = w - c(e, \theta)$, where the cost of education is $c(e, \theta) = e\theta$. Verify whether the single-crossing condition holds. Describe *all* the equilibria of this model (education choices, wages, beliefs).
2. As above, workers have productivity $\theta \in \{\theta_L, \theta_H\}$, $\theta_H > \theta_L$, but there is an equal probability of each type. Assume the cost of education is *the same* for both worker types, $c(e) = e$. Suppose the utility of worker θ who is paid wage w and undertakes education e is $U(w, e, \theta) = \theta w - e$. Is there an equilibrium where different types of workers choose different education levels? If there is, please describe all such equilibria. If not, please explain why. Show your work.