

UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MACROECONOMICS THEORY Preliminary Exam

July 29, 2004

9:00 am - 12:30 pm

INSTRUCTIONS

- Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers,. To complete the label, write:
 - (1) your assigned number
 - (2) the number of the question you are answering
 - (3) the position of the page in the sequence of pages used to answer the questions.

Example: "#001"

Question 1, page 2 of 3

- Do not answer more than one question on the same page !
When you start a new question, start a new page.
- DO NOT write your name anywhere on your answer sheets!
After the examination, the question sheets and answer sheets will be collected.
- Please DO NOT WRITE on the question sheets.
- Please solve any two of the three problems.
- You are not allowed to use notes, books, calculators, or colleagues.

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.
- Please return unused portions of the yellow tablet & question sheets.
- There are 6 pages in the exam – please make sure you have all of them.
- Good luck!

1 Problem # 1: Government Spending, Congestion and Growth (100 points)

Consider an economy in which individuals have preferences given by:

$$U_i = \sum_{t=0}^{\infty} \beta^t u_i(c_{it}) \quad 0 < \beta = \frac{1}{1+\rho} < 1,$$

where each u_i is twice differentiable strictly increasing and strictly concave. (If necessary, assume that the Inada conditions hold). Individuals differ in terms of their levels of initial wealth and, potentially, their labor endowment. Assume that the aggregate amount of labor in the economy is normalized to one.

The production set of firm j depends on its choices of capital and labor, k_j and n_j , the aggregate level of a public good, G , and an aggregate measure of net output, $F(K, N)$, where (K, N) is the vector of economy-wide levels of capital and labor (i.e. the sum across all firms). Specifically, the production set of each firm is given by

$$c_j + x_j \leq F(k_j, n_j) m \left(\frac{G}{F(K, N)} \right).$$

The production function F is homogeneous of degree one, increasing in each argument, concave and twice differentiable. In addition,

$$\begin{aligned} \lim_{K \rightarrow 0} F_k(K, 1) &= \infty, \\ \lim_{K \rightarrow \infty} F_k(K, 1) &= 0. \end{aligned}$$

The function m —which is viewed as a constant by each individual firm— is increasing, continuously differentiable, and it satisfies

$$\begin{aligned} \lim_{x \rightarrow 0} m(x) &= m_L > 0, \\ \lim_{x \rightarrow \infty} m(x) &= \infty. \end{aligned}$$

This function captures the idea that a higher level of the public good increases productivity of all firms. However, this positive effect is subject to congestion externalities: For a given level of G , an increase in ‘private’ output $F(k, n)$, has a negative impact on the productivity of each individual firm.

The capital stock in this economy evolves according to

$$K_{t+1} = (1 - \delta)K_t + X_t,$$

with total output satisfying

$$C_t + X_t + G_t \leq \sum_{j=1}^J F(k_j, n_j) m \left(\frac{G}{F(K, N)} \right),$$

where capital letters denote the economy wide aggregates of consumption, C , investment, X , capital, K , and the level of the public good, G . As indicated before $K = \sum_{j=1}^J k_j$, and $N = \sum_{j=1}^J n_j$.

In order to finance purchases of G , the government levies taxes on capital (which is owned by the households and rented out to firms) and labor income at the rate τ .

In this economy there are at least two ways in which one could define an equilibrium. One approach is to take the tax rate as given, and to let the economy determine endogenously the level of G . Denote this as “Policy I.” An alternative approach is to fix G (at some feasible level), and to let the tax rate adjust so as to satisfy the government budget constraint. Denote this as “Policy II.”

1. (30 points) Consider an economy in which the government selects “Policy I” (fixed τ). Argue that there exists a steady state and it is unique.
2. (30 points) Assume that $m(x) = x^\phi$, for $0 < \phi < 1$. Assume that, as before, fiscal policy is of the “Policy I” (fixed τ) variety. Go as far as you can describing the relationship between the steady state level of output at the tax rate τ ? Is it true that high tax countries are also low output countries? Explain your findings.
3. (40 points) Consider now the case in which a country chooses “Policy II” (fixed G). Argue that, for general $m(x)$ functions, if a steady state exists, it is not unique. Explain your results. In particular, go as far as you can providing intuition about the difference between the properties of the steady states under policies I and II.

Problem # 2 (100 points)

I) Consumption/Saving with Preference Shocks (40 points)

Consider an individual with preferences

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{z_t c_t^{1-\sigma}}{1-\sigma} \right], 0 < \sigma < 1,$$

where the preference shock $\{z_t\}$ is iid and takes values in a finite set $Z = \{\widehat{z}_1, \widehat{z}_2, \dots, \widehat{z}_n\}$, and restrict $\sigma < 1$. Assume that the shocks are ordered so that $\widehat{z}_1 < \widehat{z}_2 < \dots < \widehat{z}_n$ and let $P = [p_{ij}]$ be the transition matrix. The individual has a constant endowment $\omega > 0$, so the law of motion for wealth is

$$A_{t+1} = (1+r)(A_t + \omega - c_t),$$

for all t . Assume that no borrowing is allowed, so that $A_t \geq 0$, for all t . Assume that there exists an upper bound for assets \bar{A} .

- Formulate the Bellman equation for the above problem. (5 points)
- Find the first order and envelope conditions. (5 points)
- Show that the optimal consumption policy is increasing in A . (10 points)
- Show that the optimal consumption policy is increasing in z if the shocks are iid with probabilities $p = (p_1, p_2, \dots, p_n)$. (10 points)
- Does the result in part d go through in general (i.e. if the shocks were not iid)? Explain. (10 points)

II) Ergodic Sets (20 points)

Consider the transition matrix

$$\Pi = \begin{bmatrix} \Pi_1 & 0 \\ 0 & \Pi_2 \end{bmatrix},$$

where Π_1 and Π_2 are $k \times k$ and $(l-k) \times (l-k)$ Markov matrices.

- What is (are) the ergodic sets. (5 points)
- What is $\lim_{n \rightarrow \infty} \Pi^n$. (5 points)
- What is (are) the invariant distributions? (10 points)

III) Optimal Inventory Investment (40 points)

Robinson Crusoe lives alone. Each period he receives a random endowment ω . The endowments are iid over time, taking values on the interval $[\underline{\omega}, \bar{\omega}]$ where $\underline{\omega} \geq 0$. The continuous, strictly positive density function $f(\omega)$ describes the distribution of endowments.

Goods are storable, but require some effort. Storing $y > 0$ units of goods requires effort that reduces his utility by $\phi(y)$, where the cost function ϕ is strictly increasing, strictly convex and continuously differentiable, with $\phi(0) = \phi'(0) = 0$.

In each period, he decides how much to consume, c_t and how much to store, y_t . His period utility function is $u(c_t) - \phi(y_t)$, where u is continuous, strictly increasing, strictly concave and continuously differentiable. Crusoe discounts future utility by the constant $0 < \beta < 1$.

a. Write down the Bellman equation for Crusoe's problem. Can you write it down as a single state dynamic programming problem? (10 points)

b. Characterize the value function and policy function as sharply as you can. (30 points)

Problem # 3 - An open economy sticky price model (100 points)

Variable definitions: Superscripts h and $*$ are used to denote variables in a “home” country (say, Europe) and foreign country (say, the U.S.). Differences between the two are expressed without a superscript. Upper case letters denote levels, lower case letters denote logs:

Y_t^h, Y_t^* : real output (=consumption) levels in the two countries, with $y_t^h = \log(Y_t^h)$, $y_t^* = \log(Y_t^*)$, $y_t = y_t^h - y_t^*$
 P_t^h, P_t^* : price levels, with $p_t^h = \log(P_t^h)$, $p_t^* = \log(P_t^*)$, $p_t = p_t^h - p_t^*$
 π_t^h, π_t^* : inflation, $\pi_t^h = p_t^h - p_{t-1}^h$, $\pi_t^* = p_t^* - p_{t-1}^*$, $\pi_t = \pi_t^h - \pi_t^*$
 i_t^h, i_t^* : net nominal interest rate on one period nominally riskless debt, $i_t = i_t^h - i_t^*$
 S_t = nominal exchange rate (e.g., Euros/dollar), $s_t = \log(S_t)$
 Q_t = real exchange rate = $S_t P_t^* / P_t^h$, $q_t = \log(Q_t) = s_t + p_t^* - p_t^h = s_t - p_t$.

Note that larger values of s_t and q_t denote a weaker (depreciated) Euro.

Per period utility for consumers is

$$(Y_t^h)^{1-\sigma}/(1-\sigma) \text{ (home), } (Y_t^*)^{1-\sigma}/(1-\sigma) \text{ (foreign)}$$

with the same σ in each country. The subjective discount rate for consumers in each country is β . For parts A and B of the question, assume there is no uncertainty.

- (10) A. Explain why “uncovered interest parity” should hold:

$$(3.1) \quad 1 + i_t = (1 + i_t^*) S_{t+1} / S_t$$

- (20) B. (Open economy IS curve.)

- (5) B1. What is the condition that the consumer in the home country cannot be made better off by consuming one fewer unit today, investing to get gross nominal return $1 + i_t$, and consuming the proceeds tomorrow? (No need to derive formally, but feel free to posit a budget constraint and formal maximization problem if you wish to do so.)

- (10) B2. Show that it follows that

$$(3.2) \quad \beta (Y_{t+1}^h / Y_t^h)^{-\sigma} (P_t^h / P_{t+1}^h) = \beta (Y_{t+1}^* / Y_t^*)^{-\sigma} (P_t^* / P_{t+1}^*) (S_t / S_{t+1})$$

- (5) B3. Show that (3.2) implies the following “open economy IS curve:”

$$(3.3) \quad y_t = \sigma^{-1} q_t + \text{constant}$$

where $y_t = y_t^h - y_t^*$ and the constant depends on the initial (period 0) values of y^h, y^* and q .

- (70) C. (Dynamics.) Now add on shocks and uncertainty, dropping constants and using the approximation $\log(1+x) \approx x$ for small x . The interest parity and IS curves (3.1) and (3.3) become

$$(3.4) \quad \begin{aligned} i_t^h - i_t^* &\equiv i_t = E_t \Delta s_{t+1} + u_{rt} \Rightarrow \\ i_t - E_t \pi_{t+1} &= E_t \Delta q_{t+1} + u_{rt} \end{aligned}$$

$$(3.5) \quad y_t = \sigma^{-1} q_t + u_{st}$$

where u_{rt} and u_{st} are unobserved shocks. In this question, u_{rt} can be considered an exchange rate risk premium, u_{st} a shock to productivity in home relative to the foreign country. Assume as well identical Calvo price setting mechanisms in the two countries, leading to

$$(3.6) \quad \pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

Assume the foreign and home countries follows monetary policy rules of the form

$$(3.7a) \quad i_t^* = \gamma_\pi \pi_t^* + \gamma_y y_t^*$$

$$(3.7b) \quad i_t^h = \gamma_\pi \pi_t^h + \gamma_y y_t^h + \gamma_q q_t$$

Note that the home but not the foreign country includes q_t in the monetary rule. Upon subtracting foreign from home monetary rule we obtain

$$(3.8) \quad i_t = \gamma_q q_t + \gamma_\pi \pi_t + \gamma_y y_t$$

Through the remainder of the question, assume that $\sigma > 0$, $0 < \beta < 1$, $\kappa > 0$, $\gamma_\pi > 1$, $\gamma_y > 0$ and $\gamma_q \geq 0$.

- (10) C1. What are possible reasons for including the real exchange rate q_t in the monetary rule (3.7b)?

Suppose as well that the shocks u_{rt} and u_{st} are i.i.d..

- (15) C2. Show that a solution to (3.4), (3.5), (3.6) and (3.8) is one in which q_t , y_t , i_t and π_t are i.i.d..
- (15) C3. Deduce and interpret the impulse response of q_t , y_t , π_t and i_t to a rise in u_{rt} . Show that the response of i_t is larger (in absolute value), and the responses of q_t , y_t and π_t smaller (in absolute value), when $\gamma_q > 0$ than when $\gamma_q = 0$. What is the intuition?
- (15) C4. Deduce and interpret the impulse response of q_t , y_t , π_t and i_t to a rise in u_{st} . Show that the responses of i_t and q_t are smaller (in absolute value), and the responses of y_t and π_t larger (in absolute value), when $\gamma_q > 0$ than when $\gamma_q = 0$. What is the intuition?
- (15) C5. An empirical paper attempted to gauge the effect on the volatility of y and π of alternative values of γ_q . The paper argued that $\gamma_q = 0$ in the sample in question. It used the model above to construct time series of the shocks. (The paper allowed shocks to (3.6) and (3.8), and allowed all shocks to be serially correlated, complications that we've ignored.) It then simulated how q , y and π would have behaved for various positive values of γ_q , holding fixed the time series for the shocks. Representative results:

γ_q	Standard deviation of		
	q_t	y_t	π_t
0.0	10.9	1.6	1.1 (actual standard deviations)
0.1	7.6	1.8	1.2 (standard deviations under hypothetical policy with $\gamma_q = 0.1$)

According to the analysis above, did shocks to u_{rt} or shocks to u_{st} dominate in the sample?