UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MICROECONOMIC THEORY

July 30, 2002

9:00 am - 1:00 pm

INSTRUCTIONS

Answer four of the following six questions. On the label at top of EACH answer page, write:

(1) your assigned number, (2) the number of the question you are answering, and (3) the number of the page in the sequence of pages used to answer the questions.

Example:

ASSIGNED # 001

Q # 1 (Pg 1 of 3)

Do not answer more than one question on the same page.

Do not write your name anywhere on your answer sheets.

Do not write on the question sheets.

After the examination, the question sheets, and answer sheets will be collected.

Read the questions carefully. If a question seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that are to the point of the question and that demonstrate your command of the relevant economic theory.
QUESTION 1. Consider the following public goods provision game. We have $N \geq 2$ periods and $N$ players. In the first period, player 1 can provide the good or not. If he does so, he gets a payoff of $v - c$, each of the other players gets $v$, and the game ends. Assume $v > c > 0$. If 1 does not provide the good, we move to period 2. Now player 2 can provide the good or not. If he does so, 2 gets $v - c$, all the other players get $v$, and the game ends. If he does not, we move to period 3 where it is player 3’s opportunity to provide the good, etc. If we go through all $N$ periods with the good never provided, all players get a payoff of 0. The players discount the future at rate $\delta \in (0, 1)$. Summarizing the payoffs, then, if player $i$ provides the good in period $t$, his payoff is $\delta^{t-i}(v - c)$ and each other player’s payoff is $\delta^{t-i}v$. If no one ever provides the good, all players get 0. Consider the pure strategy subgame perfect equilibria of this game.

First, suppose that

$$1 - \frac{c}{v} < \delta < \left(1 - \frac{c}{v}\right)^{1/2}.$$ 

In what period is the public good provided? How does your answer change if instead you assume

$$\left(1 - \frac{c}{v}\right)^{1/2} < \delta < \left(1 - \frac{c}{v}\right)^{1/3}.$$ 

Give as general a statement as you can regarding how the magnitude of $\delta$ determines the period in which the public good is provided.
QUESTION 2. Consider a monopolist selling to a consumer who has utility \( U(\theta, q) = \theta V(q) - T \) if he consumes quantity \( q \), pays \( T \), and his taste parameter is \( \theta \). Assume that (unless otherwise stated) \( V''(q) > 0 \), and \( V''(q) < 0 \). Assume further that the cost to the monopolist of producing \( q \) units is \( c(q) = cq \).

Assume that the consumer is of type \( \theta_L \) with probability \( \alpha \) and \( \theta_H \) with probability \( 1 - \alpha \).

(2a) What is the socially efficient solution?

(2b) What would be the optimal solution for the monopolist if she could observe \( \theta \) (and discriminate accordingly) and could charge any nonlinear tariff \( T(q) \)?

(2c) Suppose now that the monopolist can discriminate across types of buyers but can only charge linear prices (namely, \( P(q) = pq \)) to each type. What would be the optimal solution for the monopolist? Who is better off in comparison to (ii)? Is there any sense in which the allocation in (ii) is better?

(2d) Suppose now that the monopolist can use non-linear tariffs (as in (ii)) but does not observe \( \theta \). Write down the conditions that must be satisfied at the optimal solution and interpret them. What type of distortion is introduced and why does the monopolist use it? (After you set up the problem, show which of the constraints, which you should properly name, are not binding. Once you get rid of the non-binding constraints the first order conditions that characterize the simplified problem should be simple to present).

(2e) Now assume that \( V(q) = (1 - (1 - q)^2)/2 \) and find the optimal quantity for both type \( \theta_L \) and \( \theta_H \).
QUESTION 3 Consider a two-person, two-good pure exchange economy (with free disposal). The two consumers have identical preferences, with utility function

\[(x, y) = -(x - a)^2 - (y - a)^2\]

where \(x\) denotes consumption of one good and \(y\) denotes consumption of the other good, and \(a\) is a number that is greater than \(\frac{1}{2}\), but less than 1. One consumer is endowed with 1 unit of \(x\) and zero units of \(y\), and the other is endowed with 1 unit of \(y\) and zero units of \(x\).

1. Find the set of Pareto optimal allocations.
2. Find the offer curves.
3. Find a competitive equilibrium, and determine whether it is unique.
4. Does the first welfare theorem hold for this economy? If so, prove it. If not, give a counterexample.
5. Does the second welfare theorem hold for this economy? If so, prove it. If not, give a counterexample.
QUESTION 4. Answer the following five questions:

(4a) Suppose that it costs a theater $2 per customer to show a movie and $1 apiece per box of popcorn served. The theater faces consumers who attach a valuation of 5 to seeing the movie and attach valuation 5-q to consuming their qth box of popcorn. Each consumer's total valuation is the sum of his valuation from watching the movie and consuming popcorn. The theater can choose a price for movie tickets and a price for each box of popcorn it sells. Which prices will it set to maximize its profits?

(4b) Now suppose that proportion $\alpha$ of the consumers are as described part (1a), and proportion 1-$\alpha$ have a valuation of $3 from watching the movie and place no valuation on popcorn. The theater cannot identify customers' valuations and must charge all consumers the same price. What prices will it set to maximize profits? How does your answer depend on the value of $\alpha$?

(4c) Consider again the situation of part (4b). Suppose the theater can open a "popcorn club", in which customers pay a price $P$ to purchase a box, which they can then have filled with popcorn, whenever they would like, at price $p$. As before, they can set also set a price for a movie ticket, but must charge all consumers the same price. What prices will they set?

(4d) Now suppose that proportion $\alpha$ of the consumers place a valuation of 5 on seeing the movie and no value on popcorn, while proportion 1-$\alpha$ have a valuation of $3 from watching the movie and attach valuation 5-q to consuming their qth box of popcorn. The theater cannot identify customers' valuations and must charge all consumers the same price. What prices will it set to maximize profits? How does your answer depend on the value of $\alpha$?

(4e) Movie theaters are notorious for charging high prices for their popcorn. However, they charge nothing for access to the bathroom. Why?
QUESTION 5. Consider the following normal form game $G$:

$$
\begin{array}{ccc}
 & a & b & c \\
A & 0,3 & 8,5 & 4,2 \\
B & 2,6 & 6,3 & 4,5 \\
C & 4,4 & 0,3 & 0,3 \\
\end{array}
$$

(5a). (2 points) Describe the rationalizable strategies for $G$.

(5b). (2 points) Compute all Nash equilibria of $G$.

Now consider $G^\infty(\delta)$, the infinitely repeated version of $G$, where $\delta \in (0, 1)$.

(5c). (2 points) For what values of $\delta$ are there Nash equilibria of $G^\infty(\delta)$ in which $(C, a)$ is played in all periods on the equilibrium path? Describe a strategy profile of $G^\infty(\delta)$ which is a Nash equilibrium for all of the values of $\delta$ you have specified.

(5d). (2 points) For what values of $\delta$ are there Nash equilibria of $G^\infty(\delta)$ in which $(B, c)$ is played in all periods on the equilibrium path? Describe a strategy profile of $G^\infty(\delta)$ which is a Nash equilibrium for all of the values of $\delta$ you have specified.

(5e). (2 points) For what values of $\delta$ are there Nash equilibria of $G^\infty(\delta)$ in which $(B, b)$ is played in all periods on the equilibrium path? Describe a strategy profile of $G^\infty(\delta)$ which is a Nash equilibrium for all of the values of $\delta$ you have specified.
QUESTION 6. Consider a competitive economy where there are two technologies for producing output

(1) \[ y = \min\{k^{1/2}, x\} \]

(2) \[ y = \min\{ak^{1/2}, bx\} \]

where \( k \) is the quantity of capital used by the firm and \( x \) is the level of pollution produced by the firm at output level \( y \). Firms rent capital at a rental rate of \( r \) and sell output at a market price of \( p \). Pollution is "freely" dumped into the environment. Assume that \( 0 < a < 1 < b \) so that technology (2) requires more capital per unit output but generates less pollution per unit output than technology (1).

The representative consumer has a utility function

\[ u(y, x) = y^A / x^{1-A}, \quad 0 < A < 1 \]

and owns \( K \) units of capital. Assume that there are an equal number of firms as there are consumers. Assume that consumers' incomes consist of the rental incomes from capital and the profits, \( J \), of the firms. Assume the profits, \( J \), of the firms are distributed equally and distributed lump sum to the consumers. Furthermore, consumers have no say in the quantity of pollution they are forced to consume.

(6a) What are the equilibrium values of \( p/r \) and \( J/r \) if all firms use technology (1)?

(6b) What are the equilibrium values of \( p/r \) and \( J/r \) if all firms use technology (2)? What can you say about the ratio of profits to rental rate in equilibrium?

(6c) Under what conditions is the equilibrium using technology (2) more efficient than the equilibrium using technology (1)? Measure "efficiency" by utility produced in equilibrium.

(6d) Is either equilibrium a Nash equilibrium? I.e. is it optimal for a firm to use technology \( i \), given that all others are using technology \( i \), \( i=1,2 \)? Assume each firm is small relative to the total number of firms so you can ignore feedback effects on equilibrium prices and rental rates caused by the actions of one firm.