Question 1

Consider a pure exchange economy with three consumers (labeled 1,2 and 3) and three goods (labeled x,y and z). Agent i's consumption vector is (x_i, y_i, z_i). Each agent is endowed with only one type of good: $\omega_1 = (1,0,0)$, $\omega_2 = (0,0,3)$ and $\omega_3 = (0,2,0)$ respectively. The consumers' preferences can be represented by utility functions, as follows:

$$U_1 = \sqrt{\frac{x_1(y_1 + z_1)}{5}}$$

$$U_2 = \sqrt{5x_2 y_2}$$

$$U_3 = \sqrt{5x_3 y_3}$$

a. Find a competitive equilibrium. Is it unique?

b. Find the set of Pareto optimal allocations.

Question 2

Micro-Prelim, Summer 2005

Consider an economy with n identical consumers. Each one of them consumes a good c and leisure l. The utility function is assumed to be concave and separable. The government sets the price of c at p_c and keeps it at that level. Each consumer is endowed with an exogenous income which is fixed at y. The total amount of the consumption good c produced by this economy is $\bar{C}$.

Each person wants to spend the entire income y on the consumption good. But suppose that $\bar{C}$ is not sufficient for all of them to do that. However, each consumer can increase her/his consumption of c by spending additional time, t_q, standing in line for the scarce supplies. The "queuing" time required for consumer i to get a given quantity of c is a function of $\bar{C}$ and of the sum of the time spent in line by all other consumers, $\sum_{j \neq i} t_q^j$. This queuing function has the following properties: (i) it increases at an increasing rate as c^i increases; (ii) it increases at an increasing rate as $\bar{C}$ increases; (iii) it increases at an increasing rate as $\sum_{j \neq i} t_q^j$ increases; (iv) the cross derivatives between these variables are positive.

Let T be the total amount of time available to each consumer and assume that $l + t_q = T$. Finally suppose that if all consumers spend an equal amount of time in line, t_q, an equal share $\frac{\bar{C}}{n}$ is allocated to each one of them.

1. Write down the maximization problem for consumer i.
2. Derive the equations characterizing an equilibrium where all the identical consumers behave identically.
3. Find the effect of an increase in $\bar{C}$ on the queuing time t_q and leisure l. Is the effect on leisure positive or negative? Discuss.
4. Are the equilibria that you have found Pareto-optimal? Discuss.

Now suppose that total output $\bar{C}$ is not fixed, but it is negatively related to the equilibrium level of t_q, since workers have to take time off from work to spend time in line. But assume unrealistically that individual income is still fixed at y.

5. Derive the equations characterizing an equilibrium where all the identical consumers behave identically.
6. Give an example in which the equilibrium is not unique and discuss the intuition.

Question 3

Summer 2005 Prelim Question

Consider a market in which their are n buyers, with characteristics $b_1 < b_2 < \cdots < b_n$ and m sellers with characteristics $s_1 < s_2 \cdots < s_m$. Begin by assuming $n > m$.

1. Suppose that if buyer of characteristic b and seller of characteristic s trade, then they create a surplus $b - s$. Identify the efficient allocation in this market.

2. Suppose that the market posts a price p, at which agents are free to trade or not. Identify the equilibrium price(s).

3. Suppose there is an auction in which each buyer and seller submits a price. The market clearing rule is that first a price p is found such that there are as many buyers bidding above p as there are sellers bidding below it. Those sellers bidding below p sell their good to those buyers bidding above p, all at a price equal to the highest buyer bid below p. (Other buyers do not buy and other sellers do not sell.) Identify equilibrium strategies in this auction. Show that these strategies are dominant. Compare the outcome to the efficient outcome.

4. Now suppose that when a buyer of characteristic b and seller of characteristic s trade, they create a surplus given by $v(b,s)$, which is increasing in b and s (note that high s now means a larger and not smaller surplus), with $d^2v(b,s)/dbds > 0$. *Retain this technology for the remainder of this question, along with the assumption that $n = m$.* Show that the surplus function is supermodular (i.e., if $b > b'$ and $s > s'$, then $v(b,s) + v(b',s') > v(b,s') + v(b',s)$.)

5. When a buyer and seller of types b and s trade, creating surplus $v(b,s)$, the amount $v(b,s) - p(b,s)$ of this goes to the buyer and $p(b,s)$ to the seller, where $p(b,s)$ can be viewed as the price at which buyer b purchases from seller s. An allocation is a specification of which buyers match with which sellers and the prices at which they trade. Identify all efficient allocations(s).

6. Say that a market outcome is stable if there is no buyer b' and seller s' who could match with one another and set a price $p(b',s')$ that would make both better off than their current match. Give sufficient conditions for stability in terms the market outcome and the pricing function $p(b,s)$. Compare your answer to that of part 5. Assume $m = n = 2$ and show that there exists a stable efficient outcome.

7. Fix an efficient outcome and the corresponding prices $p(b,s)$ for which it is stable. Is it possible that this price function could be constant in s? Is it possible that this price function could be constant in b?

Question 4

Prelim Exam 45 Minute Question
Summer 2005

I. (5) Consider the following candidate aggregate demand functions for two goods:

(1) $x_1(p_1, p_2, M) = \dfrac{M^{a_1}}{(p_2)^{b_1}}$;

(2) $x_2(p_1, p_2, M) = \dfrac{M^{a_2}(p_1 a_{21} + p_2 a_{22})}{(p_2)^{b_2}}$.

Here p_1 and p_2 are the prices of goods one and two and M is income.

Find conditions on the constants a_1, b_1, a_2, b_2, a_{21}, a_{22} for (1) and (2) to be aggregate demand functions. You are allowed to place a restriction on the domain of (p_1, p_2) in addition to the usual nonnegativity of prices if needed for economic sense.

II. (5) Consider the following candidate aggregate demand functions for two goods:

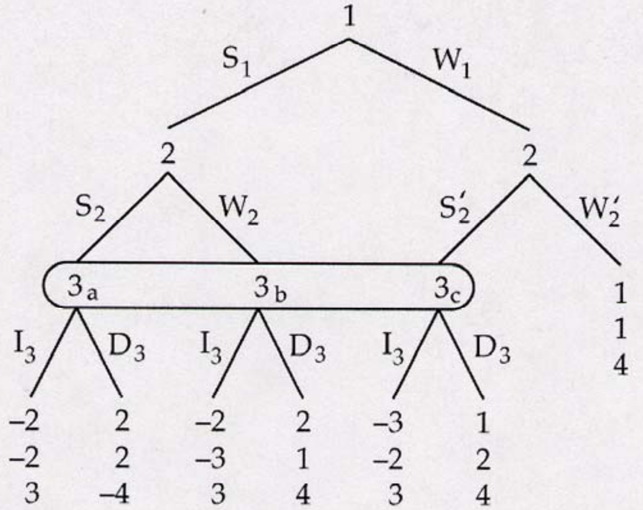
(3) $x_1(p_1, p_2, M) = \dfrac{M}{p_2}$;

(4) $x_2(p_1, p_2, M) = \dfrac{M(p_2 - p_1)}{(p_2)^2}$.

Here p_1 and p_2 are prices of goods one and two and M is income. Can (3) and (4) be the demand functions for two goods on the domain of prices, $p_2 > p_1$, for a single consumer maximizing a locally nonsatiated utility function? Justify your answer.

Question 5

Micro Prelim, July 2005



In the game Γ displayed above, players 1 and 2 are employees who are have been assigned the task of building a widget, and player 3 is their manager. Each of the two employees must choose between sleeping and working. The quality of the widget depends on the number of employees who work. If both employees work, the manager can readily observe that the widget is of high quality. If one employee works the widget is of acceptable quality, while if neither works the widget is of unacceptable quality. However, the manager cannot distinguish between the latter two cases without performing an inspection. Each employee likes sleeping better than working, and since the manager yells at them both when he opts to perform an inspection, the employees dislike this very much. For his part, the manager finds it costly to perform an inspection, but not nearly as costly as failing to inspect an unacceptable widget.

a. Find all perfect Bayesian equilibria and sequential equilibria of Γ in which all players choose pure strategies.

b. In Γ player 2 moves after observing player 1's choice. Let Γ_s be the modification of Γ in which player 2 does not observe player 1's choice before she herself moves. Find all perfect Bayesian equilibria and sequential equilibria of Γ_s in which all players play pure strategies. If the manager is allowed to decide whether player 2 gets to observe player 1's choice, which arrangement would he select? What if player 1 is given this choice? What about player 2?

c. Explain intuitively why the sequential equilibrium you found in part (a) is no longer a sequential equilibrium in part (b).

d. Find all sequential equilibria, pure and mixed, of the original game Γ.

Question 6

Suppose a parent and a child play the following game. First, the child takes an action, A, that produces income for the child $I_c(A)$ and income for the parent, $I_p(A)$. Second, the parent observes the incomes I_c and I_p and then chooses a bequest, B, to leave to the child. The child's payoff is $U(I_c + B)$; the parent's payoff is $V(I_p - B) + \beta U(I_c + B)$, where $\beta > 0$. Assume that the income functions $I_c(A)$ and $I_p(A)$ are strictly concave and are maximized at $A_c > 0$ and $A_p > 0$, respectively. The bequest B can be positive or negative, and the utility functions $U(x)$ and $V(x)$ are strictly increasing and strictly concave.

a) Is the subgame perfect equilibrium level of effort A Pareto optimal? Prove your answer.

b) Assuming $V = U$ and $\beta < 1$, under what conditions will the equilibrium bequest be positive?