

UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MICROECONOMIC THEORY Prelim Exam

July 25, 2006

9:00 am - 1:00 pm

INSTRUCTIONS

- Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers,. To complete the label, write:

(1) your assigned number

(2) the number of the question you are answering

(3) the position of the page in the sequence of pages used to answer the questions.

Example:	
MICRO THEORY	7/25/06
ASSIGNED # _____	
Qu # <u> 1 </u> (Page <u> 2 </u> of <u> 4 </u>):	

- **Do not answer more than one question on the same page !**
When you start a new question, start a new page.
- **DO NOT write your name anywhere on your answer sheets!**
After the examination, the question sheets and answer sheets will be collected.
- **Please DO NOT WRITE on the question sheets.**
- Please solve any four of the six problems.
- You are not allowed to use notes, books, calculators, or colleagues.

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well-organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- Please return unused portions of the yellow tablet & question sheets.
- There are 7 pages in the exam – please make sure you have all of them.
- Good luck!

Question 1

Question 1. A market has $n > 2$ firms that costlessly produce a homogeneous product. The market has a continuum of measure 1 of consumers, each with an inverse demand curve for the product given by $p = 1 - x$, where p denotes price and x quantity. Hence, a consumer purchases quantity $x = 1 - p$ of the good when the price is p .

- 1.1 Identify the monopoly price and quantity in this market. Identify the equilibrium price and quantity in a Cournot (simultaneous quantity-setting) oligopoly. Explain how the latter depends on the number of firms.
- 1.2 Suppose that firms first simultaneously choose price, with firm i 's price denoted by p_i , $i = 1, \dots, n$. Each consumer then chooses a firm, without being able to observe firms' prices or distinguish firms in any way. Upon choosing a firm, the consumer learns that firm's price (and only that firm's price). The consumer can then either purchase (the quantity given by the consumer's inverse demand function, given the firm's price) from that firm (in which case the consumer then ceases activity) or can choose another firm. In the latter case, the consumer incurs a cost $c > 0$, loses all contact or knowledge of the first firm it encountered, and chooses a new firm from the set of firms $\{1, \dots, n\}$. (Notice that the consumer may end up choosing its original firm again, an unrealistic assumption made in order to simplify the problem by making it stationary.) The consumer then observes the price at its new firm and can either purchase (the quantity given by the consumer's inverse demand function, given the firm's price) from that firm (in which case the consumer then ceases activity) or can choose another firm. In the latter case, a cost $c > 0$ is again incurred, and the process proceeds as before. The consumer continues (without discounting) until purchasing. Identify the equilibrium firm prices and equilibrium consumer behavior. Be precise and explain why this is an equilibrium (and the only equilibrium). How does the equilibrium depend upon the number of firms n and the cost c ?
- 1.3 Repeat your answer to part [1.2], now under the assumption that $c = 0$. Compare your answers to parts [1.2] and [1.3] and interpret the difference.
- 1.4 Now return to part [1.2]. Suppose that firms $2, \dots, n$ are expected by consumers to set the equilibrium prices calculated in part 1.2. How much would firm 1 pay to secretly make its price available to consumers before they make their choices, assuming that this is the only information consumers receive and that other firms do not know consumers have this information (and hence are expected to continue with their part-1.2 prices)? What is the equilibrium outcome if each firm has the option of costlessly making its price known to consumers (and everyone knows that this is the case)? In light of your answers, how might firms feel about informational innovations that make it harder or easier for consumers to ascertain firms' prices?

Question 2

Consider an industry of N identical competitive (i.e. price taking) firms, each maximizing profits,

$$(1) \quad pq - c(q, T) - F,$$

where $T > 0$ is the level of technology, p is price, q is quantity produced by each firm and $c(q, T)$ is the variable cost function and $F > 0$ is fixed cost. Let aggregate demand be given by $D(p)$. Then short run equilibrium for N fixed is given by

$$(2) \quad D(p) = N q^*(p, T),$$

where

$$(3) \quad q^*(p, T) = \operatorname{argmax}\{pq - c(q, T) - F\} = \operatorname{argmax}\{pq - c(q, T)\},$$

where the argmax is taken over q . Assume c is twice continuously differentiable, convex, and increasing in q . Assume the derivative of the demand function, $D'(p) < 0$ for all $p > 0$, unless D is perfectly inelastic. It is enough to do all exercises below for small changes in T . Point values are in parentheses beside each question number.

I. (1.5 points) Use the First and Second Order Necessary Conditions for profit maximization and equation (2) to locate a necessary and sufficient condition on the variable cost function, $c(q, T)$, for short run equilibrium price to fall and short run equilibrium quantity to increase when T is increased. Prove your assertion. Interpret the condition in economic terms.

II. (2.5 points) Let the long run take place, i.e. firms enter so that profits are driven to zero. Assume demand is perfectly inelastic at $D(p) = Q$ fixed. Find a sufficient condition on variable cost for long run equilibrium q to rise when T rises. What happens to long run equilibrium N when your sufficient condition holds. You may treat N as a continuous variable. Prove your assertion.

III. (4 points) Let firms enter so that profits are driven to zero. Assume $D'(p) < 0$. Continue to assume $F > 0$.

A. Find expressions for the impact of an increase of T on long run equilibrium price, long run equilibrium quantity and the long run equilibrium number of firms. (You may assume the number of firms is a continuous and differentiable quantity).

B. Assume the variable cost function decreases as T increases. Now specialize and assume the variable cost function is of the separable form

$$(4) \quad c(q, T) = c_1(q) c_2(T), \quad c_1' > 0, \quad c_1'' > 0, \quad c_2' < 0.$$

Locate necessary and sufficient conditions for N to fall when T increases at a long run equilibrium where $D'(p) = 0$, i.e. at a long run equilibrium where D is locally perfectly inelastic. Give an economic explanation of your findings and prove any assertions you make.

IV. (2 points) Assume short run equilibrium where price is high enough so that firm profits are positive. Locate sufficient conditions on the separable variable cost function $c_1(q) c_2(T)$ for short run equilibrium profits per firm to fall at a short run equilibrium where $D'(p) = 0$ to fall as T increases. Give an economic explanation for your findings and prove any assertions you make.

Question 3

Alice and Bob play a game Γ that proceeds as follows. To begin the game, Alice spins a *uniform*(0, 1) spinner. After seeing the result of this spin, Alice has the option of spinning again. Next, Bob, having observed Alice's results, spins the spinner. After seeing the result of his spin, he too has the option of spinning again.

The winner of the game receives a payoff of 1, and the loser a payoff of 0. The winner is determined as follows: If the total of Alice's (one or two) spins is above 1, then Bob is the winner. If Bob's total is above 1 and Alice's is not, then Alice is the winner. If neither player's total is above 1, then the player with the higher total is the winner. (In the event of a tie, Alice wins.)

1. What are Alice and Bob's pure strategy sets in $G(\Gamma)$, the reduced normal form of Γ ? Be precise. Also, describe in words a game Γ' that has the same reduced normal form as Γ , but in which Bob moves before Alice.

2. Find all subgame perfect equilibria of Γ . (Express all numerical values in your solution at two decimal points of accuracy.) In each equilibrium, what is Alice's worst possible first spin?

3. Is there a Nash equilibrium of Γ whose payoffs differ from those of all subgame perfect equilibria of Γ ? Give an example of such an equilibrium, or prove that no such equilibrium exists. (If you like, you may consider a version of Γ in which the spinner is uniform on a discrete set of outcomes—for instance, the set $\{.05, .10, .15, \dots, 1\}$.)

Consider an economy with n workers and a large mass of potential firms. The following problem is a static problem. At the beginning of the period, a firm decides whether or not to create a vacancy at cost c . Given that a total of m firms create vacancies, each firm is matched with a worker with probability $A\theta^{-\alpha}$, where $\theta = m/n$ and $0 < \alpha < 1$. Conversely, a worker is matched with a firm with probability $A\theta^{1-\alpha}$. Think of these expressions for large m and n , and for a value of A that makes the expressions fall in the $[0, 1]$ interval. (For small values of m and n we would need to modify the expressions, but this is irrelevant to the questions below.) There is free entry: total job creation m is determined so that the expected profit from creating a vacancy is zero. If a worker is not matched with a firm, she is unemployed with income zero.

For simplicity assume that firms hire only one worker. A match yields sales worth $\pi > 0$. No inputs other than the worker are required for production. The firm sets a wage w that it will pay the worker. The worker's utility function is assumed linear, so the worker maximizes income.

Whether employed or not, the worker receives a crime opportunity with probability λ during the period. Acting on a crime opportunity yields payoff $\tilde{p} \in [0, \infty)$, which is distributed according to the cumulative distribution function $F(p) = 1 - \exp(-p)$. Committing a crime does not interfere with the worker's employment and the firm will receive sales π regardless. If the worker chooses to commit a crime, she is caught with probability a independent of the particular payoff realization. If the worker is caught, she is put in jail, in which case she foregoes cumulated earnings tw , where $t > 0$ is the length of the jail sentence. If the worker goes to jail, the firm must pay a replacement cost R to find a new worker.

1. Conditional on employment status and wage, characterize the likelihood that a worker commits a crime.
2. Determine the optimal wage choice by the firm. Make the necessary assumption so that the optimal wage choice w^* is strictly positive. Also assume that π is sufficiently high so that match profits (the firm's profits conditional on being matched with a worker) are strictly positive.
3. How is job creation affected by an increase in t ?
4. Characterize the total crime rate in the economy (the fraction of crimes relative to the number of workers).
5. Suppose that keeping a person in prison for a sentence of length t costs $z(t)$, where z is increasing in t , and view the crime payoff \tilde{p} as a welfare-neutral transfer between workers. State the social planner's problem of determining the optimal sentence length t^* , and discuss the trade-offs that the social planner faces.

Suppose that crime is a profitable activity for some people, but not for others. A criminal who is caught pays a penalty J , and a criminal who is not caught receives H . The probability of being caught is σ . The alternative to crime is a legal activity that pays x , where x is randomly distributed over the population, with distribution function F . The payoffs J , H and x are measured in utilities. People decide whether to be criminals, after seeing the realization of x , according to whether the expected utility from crime exceeds the utility from legal activities.

The population is made up of two types of people, A and B, and the proportion of A-types is λ . The distribution of the returns to legal activities may be different for the two types (with distribution functions F_A and F_B), but the payoffs J and H are the same.

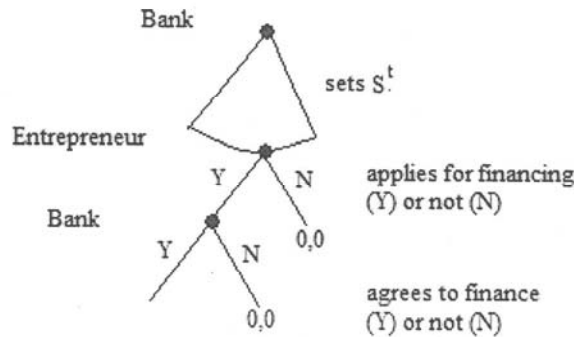
Criminals are caught when the police decide to search them (looking for stolen goods, or illegal drugs, for example). But the police have fixed resources, such that the proportion of people who are searched is s . The police may decide to search A and B types with different probabilities, σ_A and σ_B , subject to the constraint that $\lambda\sigma_A + (1-\lambda)\sigma_B = s$.

- a. If the objective of the police is to maximize the number of criminals who are caught, taking as given the number of people who have decided to be criminals, how should σ_A and σ_B be chosen?
- b. Suppose the police are required to search A and B types with equal probability, and this is known before people decide whether to be criminals. Would this increase the crime rate (relative to what it would be if the police can choose σ_A and σ_B)?
- c. If the objective of a planner is to minimize crime, and if the planner sets policy before people decide whether to be criminals, how should σ_A and σ_B be chosen?
- d. If it is observed that in practice the police are more likely to search B types than A types, is it reasonable to infer that the police are prejudiced against B types?
- e. Illustrate your answers using specific distribution functions.

Question 6

Consider an entrepreneur who needs financing from a monopolistic bank. The entrepreneur can own either a good or a bad project. Every project needs 1 unit of financing to be started up; a good project will yield profit of π_G after one period, while a bad project will yield no profit after one period. However, if the bad project is refinanced with another unit of financing, it will yield profits of π_B at the end of the second period, where $2 > \pi_G \geq \pi_B > 1$. Additional to these profits, the entrepreneur receives utility of $2\varepsilon > 0$ whenever a project makes strictly positive profits, and $-\varepsilon$ when profits are zero. Assume there is no discounting, and the probability that the project is a good one is α . The project type is the entrepreneur's private information.

a) Suppose the bank cannot commit to long-term contracts, but does observe the entrepreneur's realized profits. Financing is arranged according to the following game: Nature first draws the project type. Then the financing game depicted in the game tree below is played *twice*, where $S^t \in [0, 1]$ is the share of realized profit the bank collects as repayment in period $t \in \{1, 2\}$. (In the second period, a good project may be financed again, or a bad one refinanced). If the project is financed, the bank bears the financing cost and collects the repayment. The entrepreneur receives utility as above and the residual profit.



(i) Which projects are financed in equilibrium, and what are the equilibrium repayment schedules? (Make assumptions that ensure that some projects are financed in equilibrium).

(ii) What is the bank's expected equilibrium payoff? Would the bank be better off if it could commit to a strategy? Why? What assumption on parameter values drives this result?

b) Now suppose the bank cannot observe realized profits, and financing is arranged via a screening game. That is, the bank proposes a menu of financing decisions $f \in$

$\{Y, N\}$ and repayment schedules, $\{(f_k^1, f_k^2, S_k^1, S_k^2)\}_{k=G,B}$, whereupon the entrepreneur selects at most one schedule $(f_k^1, f_k^2, S_k^1, S_k^2)$. If a schedule is selected, the contract consisting of financing and repayments is executed; if no contract is selected, both players receive 0. In an *incentive compatible* contract, an entrepreneur of type k selects the contract designed for him $(f_k^1, f_k^2, S_k^1, S_k^2)$, $k = G, B$.

(i) What is the best incentive compatible contract the bank can propose that ensures that both types of project are financed? (Here, 'best' means profit-maximizing for the bank). What is the bank's expected payoff from this contract?

(ii) Compare your answer to (i) with your answer to a (ii) above. Explain.

(iii) Is there an incentive compatible screening contract with which the bank does better? If so, derive it.

c) Now suppose that the bank can influence a bad project's payoffs by careful monitoring during startup. Specifically, the second-period profit of a bad project is $\bar{\pi} > 1$ with probability m and 0 otherwise, where m is the bank's monitoring effort with cost $c(m) = m^2$ to the bank (first-period profits of the bad project are unchanged). Financing is again arranged via the financing game in a); monitoring occurs only in the first period, after the project has been financed but before profits (if any) accrue.

(i) If the information structure is as in a), that is, the bank does not know the project's type before profits realize, what is the bank's optimal monitoring strategy? Under which conditions on the parameters does a pooling equilibrium exist (i.e. one in which both types of entrepreneurs apply for financing)? A separating equilibrium?

(ii) When does the pooling equilibrium entail higher profits for the bank than the separating equilibrium? Why? Compare with your answer to a(ii).