UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MICROECONOMIC THEORY

January 3, 2003

9:00 am - 1:00 pm

INSTRUCTIONS

Answer four of the following six questions. On the label at top of EACH answer page, write:

(1) your assigned number, (2) the number of the question you are answering, and (3) the number
of the page in the sequence of pages used to answer the questions.

Example:

ASSIGNED # _001_

Q # ___1_____ (Pg ___1____ of ___3___)

Do not answer more than one question on the same page.

Do not write your name anywhere on your answer sheets.

Do not write on the question sheets.

After the examination, the question sheets, and answer sheets will be collected.

Read the questions carefully. If a question seems to be ambiguous, make clarifying assumptions
and state them explicitly. Aim for well organized and legible answers that are to the point of the
question and that demonstrate your command of the relevant economic theory.
QUESTION 1.

(1.1) Prove, under suitable assumptions, that any competitive equilibrium of an exchange economy is in the core.

(1.2) Give an example of a competitive equilibrium that is not in the core, and specify which of the assumptions in part (a) fails in your example.
QUESTION 2. A monopolist produces nondurable widgets at constant marginal cost zero and has no fixed costs. There is a continuum of consumers with unit demands for widgets. Each consumer’s willingness to pay for one widget is an independent draw from a strictly increasing CDF $F(\cdot)$ with support $[0,p_{\text{max}}]$ and associated density $f(\cdot)$. Assume that $2f(p) + f'(p)p > 0$ for all $p \in [0,p_{\text{max}}]$.

(2.1). Characterize the set of profit maximizing prices for the monopolist.

(2.2). Now suppose that $F(\cdot)$ is unknown but it is known that current demand is stronger than that last year. In particular, it is known that $F(p) \leq F_1(p) \forall p$, where the CDF $F_1(\cdot)$ is known. Similarly, it is known that demand is weaker now than two years ago; i.e., $F(p) \geq F_2(p) \forall p$, where $F_2(\cdot)$ is known. For $b \in \{1,2\}$, $F_b(\cdot)$ is strictly increasing, has support $[0,p_{\text{max}}]$, and has a differentiable density $f_b(\cdot)$ satisfying $2f_b(p) + f'_b(p)p > 0$ for all $p \in [0,p_{\text{max}}]$. Characterize the set of prices consistent with this information and profit-maximization by the monopolist. A graph may help.

(2.3). Is it true that a rightward shift in demand will always cause a profit maximizing monopolist to raise its price? Prove your answer and provide some concise intuition.
QUESTION 3. Consider two siblings. Alice is single and lives in Chicago. Bob is single and lives in New York. They live for two periods. In period 1 there is no uncertainty and they are endowed with an exogenous deterministic income \( y_A = \bar{y} \) (Alice) and \( y_B = \bar{y} \) (Bob). In period 2, there is uncertainty about the income realization. In particular, Alice’s income is \( \tilde{y}_A \sim N(\bar{y}, \sigma_A^2) \), where \( \sigma_A^2 \) is the variance. Bob’s income is \( \tilde{y}_B \sim N(\bar{y}, \sigma_B^2) \), where \( \sigma_B^2 \) is the variance. Denote with \( \rho \) the correlation between \( \tilde{y}_A \) and \( \tilde{y}_B \). In period 1, Alice and Bob have to decide how much to consume and how much to save in a risk-free asset with gross return \( R \), i.e. \( R \) is one plus the interest rate. In period 2, for each state of nature they consume the realization of income plus the amount saved in period 1. Alice and Bob have Constant Absolute Risk Aversion (CARA) preferences over consumption with different risk parameter, i.e.

\[
  u_A(c_A) = -\frac{\exp\left\{-r_A c_A\right\}}{r_A} \\
  u_B(c_B) = -\frac{\exp\left\{-r_B c_B\right\}}{r_B},
\]

where \( r_A > 0 \) and \( r_B > 0 \). They have identical discount factor equal to \( \beta \). Suppose for simplicity that \( \beta R = 1 \). Since they live in two different cities, they solve two different problems.

1. (3.1) Write down Alice’s problem and derive the first order conditions.

2. (3.2) Derive Alice’s savings as a function of her risk parameter, \( \bar{y}, \sigma_A^2, \beta \) and \( R \). (Hint: if \( x \sim N(\mu, \sigma^2) \) and \( t \) is a constant, then \( E[e^{tx}] = e^{\mu t + \frac{\sigma^2 t^2}{2}} \))

   Now let’s consider a different situation. Suppose that Alice and Bob both live in Chicago and they share the same apartment. They completely cooperate, in the sense that any decision is on the Pareto frontier. Moreover savings are joint. Everything else is equivalent to the previous case.

3. (3.3) Write down the Pareto problem of Alice and Bob and find the first order conditions.

4. (3.4) Assume that \( r_A = r_B = r \) and that Alice and Bob have identical decision power, i.e. identical Pareto weights. Derive TOTAL savings as a function of \( r, \bar{y}, \sigma_A^2, \sigma_B^2, \rho, \beta \) and \( R \). (Use the previous hint)

5. (3.5) Are TOTAL savings larger or smaller if they decide to live in two different cities? Provide a proof and intuition for your answer.
QUESTION 4. Consider an economy with two private goods, $x$ and $y$, and two individuals, 1 and 2. However, when person 1 consumes $x_1$ it creates an externality on person 2. Likewise, when person 2 consumes $x_2$ it creates an externality on person 1. Assume preferences are of the form

$$U_1 = y_1 + \alpha_1 \ln x_1 - \beta_1 \ln x_2$$
$$U_2 = y_2 + \alpha_2 \ln x_2 - \beta_2 \ln x_1.$$  

Assume that each person is endowed with $m$ units of good $y$ and that there is a single firm that uses $y$ as an input in the production of $x$, using a constant-returns-to-scale technology in which one unit of $y$ produces one unit of $x$.

(4.1) Find the competitive equilibrium of this economy. Prove either that it is efficient or that it is inefficient.

(4.2) Assume that the government could tax each person’s consumption of $x$ separately, i.e. choose $t_1$ to be paid by person 1 and $t_2$ to be paid by person 2. Find this first best Pigouvian tax and explain how it depends on $\beta_1$ and $\beta_2$.

(4.3) Suppose instead that the government is constrained to charge the same tax to both 1 and 2, that is, the government must choose $t_1 = t_2 = t$. Find this second-best Pigouvian tax.

(4.4) Show that the optimal second-best tax is a linear combination of the optimal first-best taxes. Give an intuitive interpretation of this.
QUESTION 5. Consider the following two-stage interaction between John the candy maker and Lois the candy retailer. At the beginning of each stage $t \in \{1, 2\}$, John chooses between making one ton of chocolate ($M_t$) and not producing anything ($D_t$). Producing costs John $1. If in either stage John does not to produce, the game ends.

If John produces in stage $t$, he gives the chocolate to Lois, who sells it to her customers, obtaining a total revenue of $8. After selling the chocolate, Lois can either split the revenue equally with John ($S_t$) or keep all of the revenue for herself ($A_t$). John is able to observe Lois' choice. If in either stage Lois keeps all of the revenue, the game ends.

(5.1) Represent this interaction as an extensive form game, and find the game's subgame perfect equilibria.

Now suppose that with probability $\frac{1}{10}$, Lois is a virtuous retailer, meaning that she only derives utility from splitting the revenue with John. With the remaining probability of $\frac{9}{10}$, Lois is a standard (self-interested) economic agent. John cannot directly observe whether Lois is virtuous or self-interested.

(5.2) Write down a new game which captures this additional detail. (Feel free to omit decision nodes which represent decisions made by a virtuous Lois.)

(5.3) Is there a perfect Bayesian equilibrium of this new game whose play path emulates the play path of some subgame perfect equilibrium of the original game? Provide either an example of such an equilibrium or a proof that no such equilibrium exists.

(5.4) Find all sequential equilibria of the new game.
QUESTION 6. Consider a consumer who derives utility from two hoods, food \((f)\) and housing \((h)\) according to the utility function \(U(f,h)\). The prices of both food and housing equal 1 throughout this problem. The consumer’s income is \(I\). Hence, the consumer’s problem is to choose \(f\) and \(h\) to solve:

\[
\begin{align*}
\text{max} & \quad U(f,h) \\
\text{subject to} & \quad f+h=I.
\end{align*}
\]

Suppose first that the utility function \(U(f,h)\) is linearly homogeneous, i.e., for any nonnegative value of \(\alpha\), we have \(U(\alpha f, \alpha h) = \alpha U(f,h)\).

(6.1) Characterize, as precisely as you can, the relationship between income \(I\) and the optimal consumption of \(f\) and \(h\).

(6.2) Characterize the relationship between the maximized value of utility and income \(I\).

(6.3) Characterize this consumer’s attitude toward risky lotteries involving different levels of income \(I\) as prizes.

Now suppose the consumer has an initial endowment of housing, denoted \(h^*\). (Assume that the value of this endowment is already contained in the consumer’s income \(I\)). The consumer must pay an adjustment cost of \(k\) if the consumer chooses any amount of housing other than \(h^*\). The consumer’s budget constraint is thus

\[
\begin{align*}
& \quad f+h = I \quad \text{if} \quad h=h^* \\
& \quad f+h = I-k \quad \text{otherwise}.
\end{align*}
\]

(6.4) Characterize, as precisely as you can, the relationship between income \(I\) and the optimal consumption of \(f\) and \(h\).

(6.5) Characterize the relationship between the maximized value of utility and income \(I\).

(6.6) Characterize this consumer’s attitude toward a lottery that puts equal probability on \(I+2\varepsilon\) and \(I-\varepsilon\) (note the asymmetry of the lottery) for two values of \(\varepsilon\), one that is arbitrarily small and one that is arbitrarily large (consistent with income being nonnegative). In particular, is the consumer risk averse, risk neutral, or risk seeking when comparing this lottery to its certainty equivalent?

It is often offered as a puzzle that consumers appear to be risk averse over relatively small-stakes gambles and risk seeking over larger gambles. For example, consumers will both purchase automobile insurance and lottery tickets. In light of your answers, how might adjustment costs in consumer goods explain this behavior?