UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MICROECONOMIC THEORY Prelim Exam

January 7, 2005

9:00 am - 1:00 pm

INSTRUCTIONS

- Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:

  (1) your assigned number  
  (2) the number of the question you are answering  
  (3) the position of the page in the sequence of pages used to answer the questions.

  Example:
  MICRO THEORY
  1/7/05
  ASSIGNED #
  Qu. # 1 (Page 2 of 4)

- **Do not answer more than one question on the same page!**
  When you start a new question, start a new page.

- **DO NOT write your name anywhere on your answer sheets!**
  After the examination, the question sheets and answer sheets will be collected.

- Please **DO NOT WRITE on the question sheets**.
- Please solve any four of the six problems.
- You are not allowed to use notes, books, calculators, or colleagues.

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well-organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- Please return unused portions of the yellow tablet & question sheets.
- There are 7 pages in the exam - please make sure you have all of them.
- Good luck!
QUESTION 1

Question:

In a two product, two input economy there are two industries that use two inputs and identical Leontief technologies,

$$q_i = \min\{z_{i1}/a_{i1}, z_{i2}/a_{i2}\}, \quad a_{ij} > 0 \quad \text{for } i = 1, 2; \quad j = 1, 2.$$ 

Let the endowment of the inputs of this economy be given by \(e = (e_1, e_2), \quad e_i > 0, \quad i = 1, 2\). Let \(a\) denote the matrix with first row \((a_{11}, a_{12})\), second row \((a_{21}, a_{22})\). Let \(p_1, p_2\) denote output prices and \(w_1, w_2\) denote input prices.

1. Assume output prices are fixed. Find an expression in terms of the matrix \(a\) and the vector \(q\) for equilibrium outputs and for equilibrium input prices. Locate sufficient conditions you need to prevent specialization.

2. If \(p_1\) increases by 50\% and \(p_2\) remains the same, find an expression for the new vector \(w\), call it \(w'\), in terms of the old vector \(w\) and the matrix \(a\).

3. Interpret the economic meaning of the assumption that the determinant of matrix \(a\) is positive. Use this assumption to sign the elements of the vector \(w' - w\).

4. Suppose a tax, \(t\), is imposed on each unit of input one, and a subsidy \(s\) is given on each unit of input two. Nothing else changes. Find expressions for the new equilibrium input prices in terms of the old equilibrium input prices. What happens to the equilibrium input prices as \(t\) increases and \(s\) increases?

5. Suppose \(e_1\) increases but \(e_2\) remains the same. Assume the determinant of matrix \(a\) is positive. Discuss what happens to equilibrium \(q\).
A market contains two types of firms, good and bad. Good firms provide an output that is a success with probability \( \phi \) and a failure with probability \( 1 - \phi \). Bad firms invariably produce failures. A success is worth 1 to consumers and a failure worth 0.

The economy operates for two periods. At the beginning of the first period, there is a unit mass of old firms, half good and half bad. There is also a unit mass of young firms, half good and half bad. Each firm sells one unit of (the costlessly produced) output at a price equal to the expected utility the output provides to consumers.

In calculating expected utility in the first period, consumers cannot tell good from bad or old from new firms, and cannot condition prices on whether their output is a success or failure. Hence, every firm receives the same first-period price.

\( (1) \) [1 point] What is the price in the first period?

At the end of the first period, all old firms die, while the young firms continue in existence in the second period. A unit mass of new firms enters the market for the second period, half good and half bad. Consumers in period 2 can observe whether a firm was young last period and had a success, young last period and had a failure, or new. Each firm's price again equals the utility consumers expect its output to provide.

\( (2) \) [1 point] What is the second-period price for each of the three types of firms?

Suppose now that each firm is identified by a name. Young firms that continue from the first period to the second must keep their names. New firms who are entering can make up a new name, or can buy a name from an old firm who is exiting and who had a success in the previous period. (To simplify things, assume that old firms who had a failure in the first period cannot sell their name.) Consumers observe whether a firm has a new name or a name that was in the market, and in the latter case whether the name had a success of failure last period, but cannot tell whether the firm bearing a name is the name's original owner or a new firm who has purchased the name.

\( (3) \) [2 points] Prove that in any equilibrium, some new firms acquire names from old firms. (Hint: Suppose not, and find a contradiction.)

\( (4) \) [6 points] Present equilibrium conditions for the second-period market in firms' output (i.e., the equilibrium prices received by each type of firm) and the second-period market for names (i.e., the price of a name purchased by a new firm, the number of names purchased, and a specification of which new firms purchase names). Present an equilibrium in which the price of a name is positive, and an equilibrium in which the price of a name is zero.
Let $G = ([1, 2], \{A_1, A_2\}, \{u_1, u_2\})$ be a two player normal form game.
Consider the following Bayesian game $\Gamma$ that is defined in terms of $G$. Let $\Theta_1$ and $\Theta_2$ be finite sets of signals. In stage 1 of $\Gamma$, players 1 and 2 observe private signals $\theta_1 \in \Theta_1$ and $\theta_2 \in \Theta_2$ that are drawn according to the joint distribution $p \in A(\Theta_1 \times \Theta_2)$. In stage 2, each player $i$, having observed his private signal, chooses a mixed action from $A_i$.

1. (1 point) Let $\sigma_1$ and $\sigma_2$ denote behavior strategies in $\Gamma$. What sorts of objects are $\sigma_1$ and $\sigma_2$? What conditions on $\sigma_1$ and $\sigma_2$ define Bayesian equilibrium in $\Gamma$?

2. (1 point) Specify a signal distribution $p$ such that the Bayesian equilibria of $\Gamma$ are identical to the Nash equilibria of $G$.

3. (3 points) Must the induced distribution over outcomes in $A_i \times A_j$ in a pure Bayesian equilibrium $(s, s')$ of $\Gamma$ also be a correlated equilibrium of $G$? Provide a proof or a counterexample.

Let $\Theta_1 = A_1$ and $\Theta_2 = A_2$, and define a new game $\Gamma'$ as follows. In stage 0 of $\Gamma'$, each player $i$ chooses a "plan" $r_i$ from $A_i \cup \{Obev\}$. In stage 1, types are determined according to the joint distribution $p \in A(\Theta_1 \times \Theta_2)$, just as in $\Gamma$. In stage 2, the decisions made in stage 0 are mechanically translated into choices in the game $G$: if in stage 0, player $i$ chose $r_i = Obev$, then in stage 2 he plays $a_i = \delta$, the action in $G$ corresponding to the signal he received in stage 1; if in stage 0 player $i$ chose $r_i \in A_i$, then in stage 2 he plays $a_i = r_i$.

4. (2 points) Fix a correlated equilibrium $\pi$ of $G$. Can we always find a signal distribution $p$ for the game $\Gamma'$ such that $\pi$ is the distribution over outcomes in a pure equilibrium of $\Gamma'$? Provide a proof or a counterexample.

5. (3 points) Must the distribution over outcomes in a pure equilibrium of $\Gamma'$ be a correlated equilibrium of $G$? Provide a proof or a counterexample.
A principal hires an agent to perform a job. The agent either "works" or "shirks." If he works, it costs disutility of \( c > 0 \) to the agent but yields the payoff of \( v > e \) to the principal (all measured in dollar terms). If he shirks, then it costs nothing to the agent but delivers zero payoff to the principal. Whether the agent works or shirks is unobservable to the principal, but the principal can find it out via an audit that costs the principal \( c > 0 \); the audit generates a verifiable evidence as to whether the agent has worked or shirked. Initially, the principal offers a contract that specifies a bonus of \( w \geq 0 \), which he pays to the agent unless she conducts an audit and it reveals that the agent has shirked. In the latter case, no bonus is paid to the agent. That is, the bonus is paid to the agent either if no audit is conducted or if the audit shows the the agent has worked. Both parties are risk neutral and the agent has zero reservation utility. Assume \( v > 4c \) and \( c > e \).

(a) Suppose that, at the time of offering a contract to the agent, the principal can commit to the probability \( a \in [0, 1] \) with which she will conduct an audit. That is, the principal first offers \((w, a)\), the agent responds by accepting or rejecting it. The game ends if the agent rejects, and both collect zero payoff. If the agent accepts it, he chooses from \( \{\text{work, shirk}\} \). And, the principal carries out the audit with probability \( a \), and \( w \) is paid unless the audit has been conducted and shows that the agent shirked. Characterize the subgame perfect equilibrium of this game, in particular, the optimal level of \((w, a)\) the principal chooses in that equilibrium.

(b) Suppose now that the principal is unable to make such a commitment. That is, the principal now simply offers \( w \geq 0 \), the agent accepts or rejects it, and, in the former case, the agent chooses from \( \{\text{work, shirk}\} \), and then the principal decides whether to audit or not. (Again \( w \) is paid unless the audit has been conducted and shows that the agent shirked.) Characterize the subgame perfect equilibrium of this game (possibly in mixed strategies), particularly the equilibrium level of bonus and audit probability and the agent's effort decision. Does the principal value the commitment power (assumed in (a))?
An economy is comprised of $N$ identical firms ($N > 1$) producing a single output by means of a single input. For $i = 1, \ldots, N$, the amount of input that firm $i$ needs in order to produce $q_i$ units of output depends not only on $q_i$ but also on the output of all firms, according to the expression

$$c(q_i, q_{-i}) = (q_i + \beta q_{-i})^\alpha q_i^\beta, \quad i = 1, \ldots, N,$$

where $q_{-i} = \sum_{j \neq i} q_j$ is the aggregate output of all other firms, and $\alpha$ and $\beta$ are parameters satisfying $\beta > 0, \alpha + \beta > 0$. Let the price of the input be normalized to one.

a) Interpret equation (A) in words for the various cases defined by the signs of $\alpha$ and of $\beta - 1$.

b) Suppose that each of the $N$ firms is an independent, price-taking, profit-maximizing decision maker. Here and in what follows, we assume that $\alpha + \beta > 1$ and that $\beta > 1$. Write firm $i$'s profit maximization problem, and its first-order equality. Rewrite the first order equality, as a function of aggregate output $q$, under the symmetry assumption that $q_i = q/N, i = 1, \ldots, N$, and solve $\alpha$ for the aggregate output, denoting its solution by $q^\star$.

c) Suppose now that the $N$ firms merge into a single firm with $N$ plants, each with the technology defined by (A) above. Find $C(q)$, defined as the minimum amount of input that the merged firm needs in order to produce $q$ units of output. In other words, $C$ is the physical cost function of the merged firm.

We assume that the merged firm is a price taker. Write its profit maximization problem for given market price of the output $p$. For which values of $\alpha, \beta$, and $p$ does the problem have a solution? Explain, and solve the profit maximization problem for these values, denoting the solution by $q^M$. For future reference, display the first order equality corresponding to an interior solution in the case $\beta > 1$.

d) Compare the supply of output in (b) and (c) for various values of the parameter $\alpha$, and discuss the dependence of aggregate supply on the organization of production in one or more firms.
Consider the following extensive form model of poker. There are two players in the game. Before cards are dealt, each player contributes a fixed amount of money, \( P \), to build the initial pot which therefore consists of an amount \( 2P \). Cards are then dealt to both players. A deck of three cards is used, and each player will be dealt one of the three cards, face down. The set of cards is given by \( S = \{1, 2, 3\} \), with a higher card number representing a greater value. Player 1 receives the first card dealt, and player 2 the second card. Assume that all card orders are equally likely to be selected.

After cards are dealt, players have the opportunity to privately look at their cards. Each player does this carefully so as not to directly reveal his card to the other player. Player 1 gets to observe his card first, and makes a choice before player 2 observes his card and makes a choice. Each player can make one of two choices, fold or bet an amount of money \( B \), which is added to the pot. If player one of the players folds, his opponent wins the pot, and the game ends. If both players bet, they then reveal their cards, and the player with the highest value card wins the pot.

a. Model this as a Bayesian game, and draw the extensive form of the game.

b. Assuming that \( B < 2P \), derive the perfect Bayesian equilibria of this game.