Reputation and TFP shocks

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How does investment in reputation respond to aggregate shocks?
Holmström (1999) applied to product markets + TFP shocks:

- Products are "experience goods" whose quality is not observable \textit{ex-ante};

- Spot markets transactions – no contracts, no contingent payments;

- Sellers of different abilities can improve quality through \textit{hidden investment};

- Sellers invest to establish reputation ("brand value").
Importance of Quality Management

Since the 1980s, Quality Management (QM) has been systematically implemented through:

- Management innovations (lean manufacturing);
- Certifications (ISO 9000, Six Sigma...).

The Centre for Economics and Business Research estimates that, in 2011, QM in the UK was responsible for: 6.02% of GDP; 1.43 million jobs.
EXAMPLE: 6 SIGMA

Initially introduced by Motorola in the 1980s:

- Assume that product quality is normally distributed;
- Ensure that there is six standard deviations between mean quality and the nearest specification limit.

A Six Sigma Process Has at Least Six Standard Deviations Between the Mean and the Nearest Spec Limit

When dealing with short-term data, this 1.5σ “buffer” is reserved for future mean-shift.
**BASIC MODEL**

Output (in efficiency units)

\[ y_t = z_t (\theta + a_t + \varepsilon_t). \]

- \( z_t \) = aggregate productivity;
- \( \theta \) = “fundamental quality”;
- \( a_t \) = effort;
- \( \varepsilon_t \) = noise.
**Results**

**Thought Experiment**: Innovation raises uncertainty about efficiency ($\theta$) and noisiness of quality ($\sigma_{\varepsilon}$).
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Findings:

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Findings:

1. A rise in $\sigma_\varepsilon$ is contractionary.
2. A rise in $\sigma_\varepsilon$ reduces responses to aggregate shocks.
3. News shocks raise output, more so for young firms.
RELATED LITERATURE

• Reputation:


• News and Business Cycles: Survey by Beaudry and Portier (2013)...
Learning without hidden action

Learning the mean of a normal distribution.

Signal is

\[ x_t = \theta + \epsilon_t. \]

\[ \epsilon_t \sim \mathcal{N} \left( 0, \sigma^2_\epsilon \right) = \text{i.i.d. shock, and } \sigma^2_\epsilon \text{ is known.} \]

\[ \theta \sim \mathcal{N} \left( m_0, \sigma^2_\theta \right) = \text{prior.} \]
Posterior updating

Observations \((x_0, ..., x_{t-1}) \equiv x^t\), posterior distribution is

\[
\theta_t \sim \mathcal{N} \left( m_t, \sigma_{\theta,t}^2 \right),
\]

where

\[
m_t = \frac{\sigma^{-2} m_0 + \sigma_{\epsilon}^{-2} \sum_{s=0}^{t-1} x_s}{\sigma_{\theta}^{-2} + \sigma_{\epsilon}^{-2} t},
\]

and

\[
\sigma_{\theta,t}^2 = \left( \sigma_{\theta}^{-2} + \sigma_{\epsilon}^{-2} t \right)^{-1}.
\]
Model with hidden action

Output (in efficiency units)

\[ y_t = z_t (\theta + a_t + \varepsilon_t), \]

\[ a_t = \text{firm’s (hidden) effort}; \]

\[ g(a_t) = \text{convex cost}. \]

Histories are \textit{public info}. 
First best

Marginal cost equals marginal returns:

\[ z_t = g'(a_t). \]

You would get it if:

1. Effort is observable;

2. Piece rates are implementable.
Learning on the equilibrium path

\[ y_t = z_t (\theta + a_t + \varepsilon_t) \]

\[ a_t^* (z_t, x^t) = \text{equilibrium action.} \]

History of signals \( x^t \equiv (x_0, ..., x_{t-1}) \) with

\[ x_t \equiv \frac{y_t}{z_t} - a_t^* (z_t, x^t) = \theta + \varepsilon_t. \]
Sample Paths

![Sample Paths Graph](image-url)
Posterior Distributions

![Graph showing prior and posterior distributions after 2 years and 5 years.](image)
Decision Problem

$$\max E_0 \left[ \sum_{t=0}^{\infty} \beta^t (R_t - g(a_t)) \right]$$

**Equilibrium:** $$(R_t (x^t, z_t), a_t^* (x^t, z_t))_{t=0}^{\infty}$$ such that

$$R_t (x^t, z_t) = E \left[ \theta \mid t, x^t, (a_s^*)_{0}^{t-1} \right] + a_t^* (x^t, z_t),$$

and

$$a_t^* (x^t, z_t) = \arg \max_{a_t \in \mathbb{R}^+} \sum_{s=t}^{\infty} \beta^{s-t} E_t [R_s - g(a_s)].$$
Belief Manipulation

Market’s posterior

\[ m_t = \frac{\sigma_\theta^{-2} m_0 + \sigma_\varepsilon^{-2} \sum_{s=0}^{t-1} x_s}{\sigma_\theta^{-2} + \sigma_\varepsilon^{-2} t}. \]

A deviation at date \( t \) raises \( \sum_{s=0}^{T-1} x_s \) by one unit for all \( T > t \).
Now

\[ \frac{\partial m_{t+s}}{\partial (\sum_{s=0}^{t-1} x_s)} = \frac{\sigma_\varepsilon^{-2}}{\sigma_\theta^{-2} + \sigma_\varepsilon^{-2} s}, \text{ for all } s > t. \] \hspace{1cm} (1)

Because effort is a function of time only, FOC at \( t \):

\[ g'(a_t^*) = \sum_{s=t+1}^{\infty} \frac{\beta^{s-t}}{\sigma_\varepsilon^2 \sigma_\theta^{-2} + s} E_t[z_s]. \] \hspace{1cm} (2)
Paths following a deviation
Belief manipulation

**CUMULATIVE OUTPUT NET OF EFFORT**

**POSTERIOR ABOUT MEAN**
DEFINITION: *Idiosyncratic volatility of* $y$:

$$\text{Var} \left\{ \frac{1}{z_t} (y_{t+1} - E_t [y_{t+1}]) \mid t, x_t, (a_s^*)_{t-1} \right\} = \sigma^2_{\theta,t} + \sigma^2_{\varepsilon},$$

Both $\sigma^2_{\theta,t}$ and $\sigma^2_{\varepsilon}$ raise output volatility but they operate differently.
Impact of news

\[ g'(a_t^*) = \sum_{s=t+1}^{\infty} \frac{\beta^{s-t}}{\sigma^2 \sigma^{-2}} + s E^t [z_s]. \]

RESULT 1: (news shocks raise output, more so for young firms)

(i) News shocks raise \( a_t^* \);

(ii) \( z_t \) affects \( a_t^* \) only through \( E^t [z_s] \) for \( s > t \).
Impact of news

$$g'(a_t^*) = \sum_{s=t+1}^{\infty} \frac{\beta^{s-t}}{\sigma^2_\epsilon \sigma^{-2}_\theta + s} E_t [z_s].$$

RESULT 2: The level of output, and its response to news, decrease with $\sigma^2_\epsilon$, and increase in $\sigma^2_\theta$. 
Volatility of revenues

**DEFINITION:** Idiosyncratic volatility of revenues $R_t$ equals $\text{Var} \{ m_t \}$ and

$$\text{Var} \{ m_t \} = \left( \frac{\sigma_{\varepsilon}^{-1}}{\sigma_{\theta,t-1}^{-2} + \sigma_{\varepsilon}^{-2}} \right)^2 = \left( \frac{\sigma_{\varepsilon}}{\sigma_{\theta}^{-2} \sigma_{\varepsilon}^2 + t} \right)^2 .$$

**RESULT 3:** $\text{Var} \{ m_t \}$ is increasing in both $\sigma_{\theta}^2$ and in $\sigma_{\varepsilon}^2$ iff

$$t > \frac{\sigma_{\varepsilon}^2}{\sigma_{\theta}^2} .$$

(3)
Revenues Volatility and $\sigma_\epsilon$

Parameter: $\sigma_\theta^2 = 10$

- $\sigma_\epsilon = 1$
- $\sigma_\epsilon = 2$
RESULT 4: The great moderation caused by the rise in idiosyncratic volatility.

PROOF: Result 2 shows that the response of output to news about TFP is decreasing in $\sigma^2_\varepsilon$. Furthermore, recall that volatility of $y_t$ is $\text{Var} \left\{ \frac{y_{t+1} - E_t[y_{t+1}]}{z_t} \right\} = \sigma^2_\varepsilon \sigma^{-2}_\theta + t$. Thus idiosyncratic volatility of $y_t$, and if $t > \sigma^2_\varepsilon \sigma^{-2}_\theta$, $R_t$ as well are increasing in $\sigma^2_\varepsilon$. 
Extension: Time-varying efficiency

Instead of remaining constant, efficiency

\[ \theta_{t+1} = \theta_t + \nu_t , \text{ with } \nu_t \sim \mathcal{N} \left( 0, \sigma_v^2 \right). \]

Then

\[
\sigma_{\theta,t}^2 = \frac{1}{\sigma_{\theta,t-1}^{-2} + \sigma_\epsilon^{-2}} + \sigma_v^2 = \frac{1 + \left( \sigma_{\theta,t-1}^{-2} + \sigma_\epsilon^{-2} \right) \sigma_v^2}{\sigma_{\theta,t-1}^{-2} + \sigma_\epsilon^{-2}},
\]

and stationary precision

\[
\sigma_{\theta}^{-2*} = \frac{1}{2} \left( \sqrt{\frac{1}{\sigma_\epsilon^4} + \frac{4}{\sigma_\epsilon^2 \sigma_v^2}} - \frac{1}{\sigma_\epsilon^2} \right).
\]

Now \( \sigma_\epsilon^2 \) has two effects: (i) Increase noise; (ii) Lower stationary precision.
Extension: Time-varying efficiency
**Stylized fact**: Durables more cyclically volatile

Usual explanation based on demand.

But it can also be explained by lower precision about $\theta$ (↑ $\sigma_\theta$)?

Decrease in relative price of durables is a proxy for rate of technological progress, and so $\sigma_\theta$. 
Demand

Utility of representative consumer

\[ U = \left[ \int_{i \in I} (y_i q_i)^{\frac{\gamma-1}{\gamma}} di \right]^\frac{\gamma}{\gamma-1}, \text{ with } \gamma > 1, \]

where \( y \) is the quantity and \( q \) is the quality. Then

\[ p_i y_i = \left( \frac{p_i}{\hat{q}_i} \right)^{1-\gamma} \frac{E}{P^{1-\gamma}}, \text{ with } \hat{q}_i \equiv E \left[ q_i^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \]

while \( P \) denotes the quality-adjusted price index

\[ P \equiv \left[ \int_{i \in I} \left( \frac{p_i}{\hat{q}_i} \right)^{1-\gamma} di \right]^\frac{1}{1-\gamma}, \]

and \( E \) is total expenditure.
Technology

1. $l_{i,t}$ workers assigned to **physical production**

\[ y_{i,t} = \exp(z_t) l_{i,t} . \]

2. $d_{i,t}$ workers assigned to **quality control**

\[ q_{i,t} = \left[ \exp(\theta_{i,t} + \epsilon_{i,t}) d_{i,t} \right]^{1/\varphi} , \text{ where } \varphi > 0, \epsilon_{i,t} \sim \mathcal{N}(0, \sigma^2_{\epsilon}) . \]

**Learning:** Quality revealed ex-post because consumers cannot observe $d$.

- When $\theta_0 \sim \mathcal{N}(m_0, \sigma^2_\theta)$, signal extraction is as in the linear model.
- Sufficient statistics: $X_{i,t} \equiv \sum_{s=1}^t x_{i,t} = \sum_{s=1}^t \ln \left( q_{i,t} / d_{i,t}^* \right)$ and $\theta_{i,t} \sim \mathcal{N}(m_t, \sigma^2_{\theta,t})$.
Optimal Quantity

\[
\max_{l \in \mathbb{R}^+} R(l; m, z) = py - l,
\]

\[
s.t. py = \left( \frac{p}{E^* [\hat{q}(m)]} \right)^{1-\gamma} \frac{E}{P^{1-\gamma}},
\]

\[
y = \exp(z) l.
\]

Since \( E^* [\hat{q}] = d^* (m)^{1/\varphi} \exp \left( \frac{m}{\varphi} + \frac{\gamma-1}{2\gamma \varphi^2} \sigma^2 \right) \), actual investment in quality \( d \) does not matter for the quantity decision.

Optimal revenues:

\[
R(m, z) = \left[ d^* (m)^{1/\varphi} \exp \left( z + \frac{m}{\varphi} + \frac{\gamma - 1}{2\gamma \varphi^2} \sigma^2 \right) \right]^{\gamma^{-1}} C(\gamma, E, P).
\]
Incentive Constraint

Effort is *not anymore* deterministic.

*Beliefs off the equilibrium:* Effort $a = \ln (d)$ and $\delta_t \equiv a_t - a^*_t$ is the deviation from equilibrium effort. Let $\Delta_t$

$$\Delta_{t+1} = \lambda_t \Delta_t + (1 - \lambda_t) \delta_t$$

with $\lambda_t \equiv \frac{\sigma_{\theta,t}^{-2}}{\sigma_{\theta,t}^{-2} + \sigma_\varepsilon^{-2}}$ and $\Delta_0 = 0$.

Then $m^a_t = m_t - \Delta_t$ and beliefs obey

$$m_{t+1} = \frac{\sigma_{\theta,t}^{-2} m_t + \sigma_\varepsilon^{-2} x_t}{\sigma_{\theta,t}^{-2} + \sigma_\varepsilon^{-2}} = m_t + (1 - \lambda_t) [\delta_t - \Delta_t + \varepsilon^a_t] .$$
Incentive Constraint

\[ V_t (m, z, \Delta) = \max_{\delta} \left\{ R (m, z) - g \left( a_t^* (m, z) + \delta \right) \right\} + \beta E_t \left[ V_{t+1} (m', z', \Delta') \right] \]

\[ s.t. \ m' = m + (1 - \lambda_t) \left[ \delta - \Delta + \varepsilon_t^a \right] , \]

\[ \Delta' = \lambda_t \Delta + (1 - \lambda_t) \delta . \]

**Necessary Condition:** Equilibrium effort \( a_t^* (m, z) \) solves

\[ g' (a_t^*) = \beta (1 - \lambda_t) E_t \left[ \frac{\partial R_{t+1}}{\partial m_{t+1}} - g' (a_{t+1}^*) \left( \frac{\partial a_{t+1}^*}{\partial m_{t+1}} - \frac{\lambda_{t+1}}{1 - \lambda_{t+1}} \right) \right] . \]
Intuition behind Necessary Condition

When effort is deterministic, i.e., $\frac{\partial a^*_s}{\partial m_s} = 0$,

$$g' (a^*_t) = \sum_{s=t+1}^{\infty} \beta^{s-t} E_t \left[ \frac{\partial R^*_s}{\partial m_s} \frac{\partial m_s}{\partial a_t} \right].$$
Intuition behind Necessary Condition

When effort is deterministic, i.e., \( \partial a_s^* / \partial m_s = 0 \),

\[
g'(a_t^*) = \sum_{s=t+1}^{\infty} \beta^{s-t} E_t \left[ \frac{\partial R_s^*}{\partial m_s} \frac{\partial m_s}{\partial a_t} \right] .
\]

Generalized condition

\[
g'(a_t^*) = \sum_{s=t+1}^{\infty} \beta^{s-t} E_t \left[ \left( \frac{\partial R_s^*}{\partial m_s} - g'(a_s^*) \frac{\partial a_s^*}{\partial m_s} \right) \frac{\partial m_s}{\partial a_t} \right] .
\]

Additional cost capture adjustments in future efforts required to sustain beliefs.
**Analytical Solution**

**PROPOSITION:** Provided that $z_{t+1} \sim \mathcal{N} \left( z_t, \sigma_z^2 \right)$, equilibrium investment is log-linear in both $m$ and $z$

$$d_t^* (m, z) = \exp \left( A_t + \frac{\gamma - 1}{1 + \varphi - \gamma} (m + z) \right),$$

with $A_t$ solving a recursive equation with terminal condition $A^* = \lim_{t \to \infty} A_t$ given by

$$e^{- \left( \frac{\gamma - 1}{1 + \varphi - \gamma} \right)^2 \frac{\sigma_m^2 + \varphi^2 \sigma_z^2}{2}} \beta = \frac{(\gamma - 1) (1 - \lambda^*)}{1 + \varphi - \gamma} \left( e^{\frac{(\gamma - 1)^2}{2\gamma \varphi^2} \sigma^2 + \left( \frac{\gamma - 1}{\varphi} - 1 \right) A^*} - 1 \right) + \lambda^*,$$

where $\sigma_{m,t} \equiv \sigma_{\varepsilon}^{-1} \left( h_t + \sigma_{\varepsilon}^{-2} \right)^{-1}$. 
Extension 2: Oligopolistic Model

The graph shows the trend of a variable $A_t$ over time with different values of $\sigma_\theta$. The x-axis represents time, and the y-axis represents the value of $A_t$. The graph includes three lines corresponding to $\sigma_\theta = 10$, $\sigma_\theta = 0.7$, and $\sigma_\theta = 0.5$. The lines demonstrate how the value of $A_t$ changes over time for each of these parameter values.
Extension 2: Oligopolistic Model

Impact of news on investment for different $\sigma_\epsilon$
Extension 2: Oligopolistic Model

$A$ as a function of time for different $\sigma_v$
Extension 2: Oligopolistic Model

\[
\begin{align*}
\text{Var}(R) & \quad \sigma_v \\
0.2 & \quad 0.51 \\
0.25 & \quad 0.50 \\
0.3 & \quad 0.49 \\
0.35 & \quad 0.48 \\
0.4 & \quad 0.47 \\
0.45 & \quad 0.46 \\
0.5 & \quad 0.45 \\
0.55 & \quad 0.44 \\
0.6 & \quad 0.43 \\
0.65 & \quad 0.42 \\
0.7 & \quad 0.41
\end{align*}
\]

\[
\begin{align*}
dR/dZ & \quad \sigma_v \\
0.2 & \quad 2.9 \\
0.25 & \quad 2.8 \\
0.3 & \quad 2.7 \\
0.35 & \quad 2.6 \\
0.4 & \quad 2.5 \\
0.45 & \quad 2.4 \\
0.5 & \quad 2.3 \\
0.55 & \quad 2.2 \\
0.6 & \quad 2.1 \\
0.65 & \quad 2.0 \\
0.7 & \quad 1.9
\end{align*}
\]
CONCLUSION

We asked how shocks interact with the reputation mechanism

RESULTS

1 News shocks raise output, more so for young firms.

2 A rise in idiosyncratic uncertainty is contractionary.

3 Reduces responses to aggregate shocks ("explains the great moderation").

4 Derived closed form solutions that could "easily" be introduced into DSGE models.