The Allocation of Talent 
and U.S. Economic Growth

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Abstract

In 1960, 94 percent of doctors were white men, as were 96 percent of lawyers 
and 86 percent of managers. By 2008, these numbers had fallen to 63, 61, and 57 
percent, respectively. We use a Roy model with occupational frictions to mea-
sure the contribution of discrimination faced by blacks and women in the labor 
market and in the acquisition of human capital to the occupational distribution 
of these groups from 1960 to 2008. We embed the Roy model in general equi-
librium to measure the aggregate effects of the change in these barriers. The 
reduction in the barriers to occupational choice facing women and blacks can 
explain 15 to 20 percent of aggregate wage growth, 90 to 95 percent of the wage 
convergence between women and blacks and white men, and 30 percent of the 
rise in women’s labor force participation from 1960 to 2008.

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Gabriel Ulyssea for excellent research assistance.
1. Introduction

Fifty years ago, there were stark differences in the occupational distribution of white men versus women and blacks. For example, virtually all doctors, lawyers, engineers, and executives and managers in 1960 were white men: 94 percent of doctors, 96 percent of lawyers, 99 percent of engineers, and 86 percent of executives and managers. In contrast, 58 percent of white women were employed as nurses, teachers, sales clerks, secretaries, and food preparers; 54 percent of black men were employed as freight handlers, drivers, machine operators, and janitors. A vast literature has documented how these gaps have narrowed since then, particularly in high-skilled occupations.\(^1\) By 2008, only 63 percent of doctors and 61 percent of lawyers were white men. Similarly, the share of women and blacks in skilled occupations increased from 2 percent in 1960 to 15 percent for women and 11 percent for black men by 2008.\(^2\)

This paper measures the aggregate effect of the changes in the occupational distribution through the prism of a Roy (1951) model of occupational choice. We assume that every person is born with a range of talents across all possible occupations and chooses the occupation where she earns the highest returns. In this framework, changes in the occupational distribution arise naturally from changes in the returns to occupational skills. For example, technical change in skill-intensive occupations will increase the returns to skills and the employment share of all groups in these occupations. In turn, differences in the occupational distribution between groups can arise from differences in the distribution of talent between groups; Rendall (2010), for example, shows that brawn-intensive occupations (such as construction) in the U.S. are dominated by men.

However, it seems unlikely that these forces can explain all the differences we see in occupational choice and how it has changed over the last fifty years. Consider the world that Supreme Court Justice Sandra Day O’Connor faced when she

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\(^1\)We will not attempt to survey this literature, but see Blau (1998), Goldin (1990), and Smith and Welch (1989) for assessments of this evidence.

\(^2\)We define skilled occupations as as executives, managers, architects, engineers, computer scientists, mathematicians, scientists, doctors, and lawyers.
graduated from Stanford Law School in 1952. Despite being ranked third in her class, the only private sector job she could get was as a legal secretary (Biskupic, 2006). Such barriers might explain why white men dominated the legal profession at that time. And the fact that private law firms are now more open to hiring talented female lawyers might explain why the share of women in the legal profession has increased dramatically over the last fifty years. Similarly, the Civil Rights movement of the 1960s is surely important in explaining the change in the occupational distribution of blacks over the last fifty years.\(^3\)

To capture these forces, we introduce barriers to occupational choice in the Roy model. We First, we allow for the possibility that each group faces different occupational frictions in the labor market. We model these frictions as a group-occupation specific “tax” on earnings that drives a wedge between the group’s marginal product in the occupation and their take home pay. One interpretation of these “taxes” is that they represent preference-based discrimination as in Becker (1957). For example, one reason why private law firms would not hire Justice O’Connor is that the law firms’ partners (or their customers) viewed the otherwise identical legal services provided by female lawyers as somehow less valuable.\(^4\) Second, we allow for group-specific frictions in the acquisition of human capital. We model these frictions as a tax for each group and each occupation on the inputs into human capital production. These human capital frictions could represent the fact that some groups were restricted from elite higher education institutions, that black public schools are underfunded relative to white public schools, that there are differences in prenatal or early life health investments across groups, or that social forces steered certain groups towards certain occupations.\(^5\)

\(^3\)See Donohue and Heckman (1991) for an assessment of the effect of federal civil rights policy on the economic welfare of blacks.

\(^4\)Consistent with the Becker (1957) interpretation of labor market frictions, Charles and Guryan (2008) show that relative black wages are lower in states where the marginal white person is more prejudiced (against blacks).

\(^5\)Here is an incomplete list of the enormous literature on these forces. Karabel (2005) documents how Harvard, Princeton, and Yale systematically discriminated against blacks, women, and Jews in admissions until the late 1960s. Card and Krueger (1992) documents that public schools for blacks in the U.S. South in the 1950s were underfunded relative to schools for white children. See Chay, Guryan and Mazumder (2009) for evidence on the importance of improved access to health care for blacks. See Fernandez (2007) and Fernandez, Fogli and Olivetti (2004) on the role of social forces in women’s
In our augmented Roy model, all three forces — relative returns to occupational skills, relative ability, and barriers to occupational choice — affect the occupational distribution of a group. To make progress analytically, we follow McFadden (1974) and Eaton and Kortum (2002) and assume that the distribution of talent follows an extreme value distribution. This assumption gives us two key results. First, we get a closed form expression relating the relative fraction of workers in different groups in an occupation to a composite of the relative talent and the relative occupational barrier of the group. We calculate this composite measure using data from the decadal U.S. Censuses and the American Community Surveys. We find that the composite of relative talent and occupational frictions improved dramatically for women and blacks in high-skilled occupations over the last 50 years, but was roughly unchanged in low skilled occupations.

Second, we get the result that the average wage gap between groups depends on a weighted average of the occupational friction where the weights depend on the talent of the group and returns to occupational skills in each occupation. Importantly, the theory predicts that only the average of these forces matter for the wage gap and that the wage gap will not be higher in an occupation where the group faces larger frictions. Intuitively, imagine that the barriers facing women in the legal profession decline. This increases the income of existing women lawyers, but it also induces less talented female lawyers to enter the legal profession. With an extreme value distribution, this quality dilution effect exactly offsets the direct effect of lower barriers on the average wage. The average wage of women rises overall but by the same amount in all occupations. Consistent with this prediction, we show that between 1960 and 2008, the relative wage of women in low-skilled occupations increased by almost exactly the same amount as that of women in high-skilled occupations.

Finally, we embed the Roy model in general equilibrium. This allows us to estimate the effect of a reduction in the barriers to occupational choice on aggregate occupational choice. Goldin and Katz (2002), Bailey (2006), Bertrand, Goldin and Katz (2010), and Bailey, Hershbein and Milleri (2012) highlight the importance of contraception and fertility choices for women’s labor market outcomes.
productivity, wages, and labor force participation. In our baseline results, 15 to 20 percent of aggregate wage growth between 1960 and 2008. Looking at the individual groups, the reduction in the frictions since 1960 boosts real wages by 39% for white women, 57% for black women, and 44% for black men, but lowers real wages by 4.3% for white men. The reduction in frictions can thus account for 90 to 95 percent of the narrowing of the wage gap between blacks and women vs. white men. In addition, about 30 percent of the rise in women’s labor force participation is attributable to the decline in occupational frictions.

The paper proceeds as follows. The next section lays out the basic model of occupational choice. We then use this framework to measure the frictions in occupational choice between blacks and women versus white men in Section 3. Section 4 embeds the occupational choice framework into general equilibrium, allowing us to explore the macroeconomic consequences of misallocation in Section 5.

2. A Model of Occupational Sorting

The economy consists of a continuum of people working in \( N \) possible occupations, one of which is the home sector. Each person possesses an idiosyncratic ability in each occupation — some people are good economists while others are good nurses. The basic economic allocation to be determined in this economy is how to match workers with occupations.

2.1. People

Individuals are members of different groups, such as race and gender, indexed by \( g \). A person with consumption \( c \) and leisure time \( 1 - s \) gets utility

\[
U = c^\beta (1 - s)
\]  

(1)

where \( s \) represents time spent on schooling, and \( \beta \) parameterizes the tradeoff between consumption and schooling.

Each person works one unit of time in an occupation indexed by \( i \) (another unit
of time — think “when young” — is divided between leisure and schooling). A person’s human capital is produced by combining time \( s \) and goods \( e \). The production function for human capital in occupation \( i \) is

\[
    h(e, s; i) = s^{\alpha_i}e^{\eta_i},
\]

We add two frictions here. The first friction affects human capital: a “tax” \( \tau_{hi} \) is applied to the goods \( e \) invested in human capital, with the tax varying across occupations and groups. We think of this tax as representing forces that affect the cost of acquiring human capital. For example, \( \tau_{hi} \) might represent discrimination against blacks or women in admission to universities, or differential allocation of resources to public schools attended by black vs. white children, or parental liquidity constraints that affect children’s health and education.

The second friction we consider can be thought of as a friction in the labor market. A person in occupation \( i \) and group \( g \) is paid a wage equal to \((1 - \tau_{wi})w_i\) where \( w_i \) denotes the wage per efficiency unit of labor paid by the firm. One interpretation of \( \tau_{wi} \) is that it represents preference-based discrimination by the employer or customers as in Becker (1957).

Consumption is equal to labor income less expenditures on education, incorporating both frictions:

\[
    c = (1 - \tau_{wi})weh(e, s) - e(1 + \tau_{hi}). \tag{3}
\]

Note that pre-distortion labor income is the product of the wage received per efficiency unit of labor, the idiosyncratic talent draw \( \epsilon \) in the worker’s chosen occupation, and the individual’s acquired human capital \( h \).

Given an occupational choice, the occupational wage \( w_i \), and idiosyncratic ability \( \epsilon \) in the occupation, each individual chooses \( c, e, s \) to maximize utility:

\[
    U(\tau^w, \tau^h, w, \epsilon) = \max_{c,e,s} (1 - s)e^\beta \quad s.t. \quad c = (1 - \tau_{wi})weh(e, s) - e(1 + \tau_{hi}) \tag{4}.
\]

This yields the following expressions for the amount of time and goods spent on
human capital:

\[ s^*_i = \frac{1}{1 + \frac{1}{\beta \phi_i}} \]

\[ e_{ig}^*(\epsilon) = \left( \frac{\eta(1 - \tau_{ig}^w) w_i s^*_i \phi_i \epsilon}{1 + \tau_{ig}^h} \right)^{\frac{1}{1-\eta}} \]

Time spent on human capital is increasing in \( \phi_i \). Individuals in high \( \phi_i \) occupations have more schooling and higher wages to compensate them for the time spent on schooling. Forces such as \( w_i, \tau_{ig}^h, \tau_{ig}^h \) do not affect \( s \) because they have the same effect on the return and on the cost of time. In contrast, these forces change the returns of investment in *goods* in human capital (relative to the cost) with an elasticity that is increasing in \( \eta \). After substituting the expression for human capital into the utility function, we get the following expression for indirect utility (conditional on choosing an occupation):

\[ U(\tau_{ig}, w_i, \epsilon_i) = \left( \frac{\eta w_i s^*_i (1 - s_i) \phi_i \epsilon_i}{\tau_{ig}^h} \right)^{\frac{\theta}{\eta}} \] (5)

Here, we use define \( \tau_{ig} \) as a “gross” tax rate that summarizes the two frictions:

\[ \tau_{ig} \equiv \frac{(1 + \tau_{ig}^h) \eta}{1 - \tau_{ig}^w}. \] (6)

### 2.2. Occupational Skills

Turning to the worker’s idiosyncratic talent, we borrow from McFadden (1974)’s and Eaton and Kortum (2002)’s formulation of the discrete choice problem. We assume each person gets an iid skill draw \( \epsilon_i \) from a Fréchet extreme value distribution for each occupation:

\[ F_{ig}(\epsilon) = \exp(-T_{ig} \epsilon^{-\theta}). \] (7)

The parameter \( \theta \) governs the dispersion of skills, with a higher value of \( \theta \) corresponding to *smaller* dispersion. We assume that \( \theta \) is common across occupations and groups. The parameter \( T_{ig} \), however, can potentially differ. Across occupations,
differences are obvious: talent is easy to come by in some occupations and scarce in others. In some occupations, it also seems reasonable to allow the distribution of talent to differ between men and women. For example, men may be relatively more endowed with physical strength, which is likely to be more valuable in occupations such as firefighting or construction.

2.3. Occupational choice

The occupational choice problem reduces to picking the occupation that delivers the highest value of $U_{ig}$. The assumption that the talent draws are iid and come from an extreme value distribution delivers the result that the highest utility can also be characterized by an extreme value distribution, a result reminiscent of those in McFadden (1974) and Eaton and Kortum (2002). The overall occupational share can then be obtained by aggregating the optimal choice across people, as we show in the next proposition.

**Proposition 1 (Occupational Choice):** Let $p_{ig}$ denote the fraction of people in group $g$ that work in occupation $i$. Aggregating across people, the solution to the individual’s occupational choice problem leads to

$$
p_{ig} = \frac{\tilde{w}_{ig}^{\theta}}{\sum_{s=1}^{N} \tilde{w}_{sg}^{\theta}} \quad \text{where} \quad \tilde{w}_{ig} = \frac{T_{ig}^{1/\theta} w_i s_i^{\phi_i} (1 - s_i)^{1 - \eta}}{\tau_{ig}}.
$$

Equation (8) says that the share of group $g$ in occupation $i$ depends on three broad sets of forces: 1) The mean talent of the group in the occupation, as captured by $T_{ig}$; 2) The price per unit of skill in the occupation, as measured by $w_i$. We think of $w_i$ as capturing the effect of technological change on the occupational distribution. For example, technological innovations in the home sector emphasized by Greenwood et al. (2005) can be viewed as a decline in $w_{i}$ in the home sector; 3) The returns to years of schooling in the occupation, as captured by $s_i^{\phi_i} (1 - s_i)^{1 - \eta}$; 4) Human capital and labor market frictions, as captured by $\tau_{ig}$. An increase in $\tau_{ig}$ lowers the share of group $g$ in occupation $i$.
Proposition 2 (Average Quality of Workers): The average quality of workers in each occupation, including both human capital and talent, is

\[ \mathbb{E} [h_i \epsilon_i] = \gamma \left[ \eta \phi_i \left( \frac{w_i}{p_{ig}} \right)^{\eta} \left( \frac{T_i}{p_{ig}} \right)^{\frac{1}{\eta}} \right] \]

(9)

where \( \gamma \equiv \Gamma(1 - \frac{1}{\eta} \cdot \frac{1}{1 - \eta}) \) is related to the mean of the Fréchet distribution for abilities.

Notice that average quality is inversely related to the share of the group in the occupation \( p_{ig} \). This captures the selection effect. For example, the model predicts that only the most talented female lawyers (such as Sandra Day O’Connor) would have chosen to be lawyers in 1960. And as the barriers faced by female lawyers declined after 1960, this induced less talented female lawyers into the legal profession and thus lowered the average quality of female lawyers.

Proposition 3 (Occupational Wage Gaps): Let \( \text{wage}_{ig} \) denote the average earnings in occupation \( i \) by group \( g \). Its value satisfies

\[ \text{wage}_{ig} = (1 - \tau_{ig}) w_i \mathbb{E} [h_i \epsilon_i] = (1 - s_i)^{-1/\beta} \gamma \left( \sum_{s=1}^{N} \tilde{w}_{sg}^{\theta} \right)^{\frac{1}{\eta}} \left( \sum_{s} \tilde{w}_{sg}^{\theta} \right)^{\frac{1}{\eta}}. \]

(10)

In turn, the occupational wage gap between any two groups is the same across all occupations. For example,

\[ \frac{\text{wage}_{ig}}{\text{wage}_{i,wm}} = \left( \frac{\sum_{s} \tilde{w}_{sg}^{\theta}}{\sum_{s} \tilde{w}_{s,wm}^{\theta}} \right)^{\frac{1}{\eta}} \left( \sum_{s} \tilde{w}_{sg}^{\theta} \right)^{\frac{1}{\eta}}. \]

(11)

The first equation of the proposition reveals that average earnings for a given group only differs across occupations because of the first term, \( (1 - s_i)^{-1/\beta} \). Occupations in which schooling is especially productive (a high \( \phi_i \) and therefore a high \( s_i \)) will have higher average earnings, and that is the only reason for earnings differences across occupations in the model. For example, occupations in which a group faces less discrimination or a better talent pool or a higher wage per efficiency unit do not yield higher average earnings. The reason is that each of these factors leads
to lower quality (i.e. lower $\epsilon$) workers entering those jobs. This composition effect exactly offsets the direct effect on earnings when the distribution of talent is Fréchet. We will test the proposition that the earnings gap between two groups will be constant across occupations.

Finally, putting together the equations for the occupational share and the wage gap together, we get the following expression for the relative propensity of a group to work in an occupation:

$$\frac{p_{ig}}{p_{i,wm}} = \frac{T_{ig}}{T_{i,wm}} \cdot \left( \frac{\tau_{ig}}{\tau_{i,wm}} \right)^{-\theta} \left( \frac{\text{wage}_g}{\text{wage}_{wm}} \right)^{\theta(1-\eta)}$$

(12)

This equation states that the propensity of a group to work in an occupation (relative to white men) depends on three terms: the relative mean talent in the occupation (arguably equal to one for many occupations), the relative occupational friction, and the average wage gap between the groups. Equation (12) says that with data on occupational shares and wages, we can measure a composite of the relative mean talent and occupational friction between groups. This will be the key equation we take to the data.

3. **Empirically Evaluating the Occupational Sorting Model**

3.1. **Data**

We use data from the 1960, 1970, 1980, 1990, and 2000 Decennial Censuses as well as data from the 2006-2008 American Community Surveys (ACS) for all analysis in the paper. When using the 2006-2008 ACS data, we pool all the years together and treat them as one cross section.\textsuperscript{6} We make only four restrictions to the raw data when constructing our analysis samples. First, we restrict the analysis to only include white men (wm), white women (ww), black men (bm) and black women (bw). These

\textsuperscript{6}Henceforth, we refer to the pooled 2006-2008 sample as the 2008 sample. A full description of how we process the data, including all the relevant code, is available at [http://faculty.chicagobooth.edu/erik.hurst/research/chad_data.html](http://faculty.chicagobooth.edu/erik.hurst/research/chad_data.html).
will be the four groups we analyze in the paper.\textsuperscript{7} Second, we restrict the sample to include only individuals between the ages of 25 and 55 (inclusive). This restriction helps to focus our analysis on individuals after they finish schooling and prior to considering retirement. Third, we exclude individuals on active military duty. Finally, we exclude individuals who report their labor market status as being unemployed (i.e., not working but searching for work). Our model is not well suited to capture transitory movements into and out of employment. Appendix Table A1 reports the sample size for each of our six cross sections, including the fraction of the sample comprised of our four groups.\textsuperscript{8}

A key to our analysis is to use the Census data to create a consistent set of occupations over time. We treat the home sector as a separate occupation. Anyone in our data who is not currently employed or who is employed but usually works less than ten hours per week is considered to be working exclusively in the home sector. Those who are employed but usually work between ten and thirty hours per week are classified as being part-time workers. We split the sampling weight of part-time workers equally between the home sector and the occupation to which they are working. Individuals working more than thirty hours per week are considered to be working full-time in an occupation outside of the home sector.

For our base analysis, we define the non-home occupations using the roughly 70 occupational sub-headings from the 1990 Census occupational classification system.\textsuperscript{9} We use the 1990 occupation codes as the basis for our occupational definitions because the 1990 occupation codes are available in all Census and ACS years since 1960. We start our analysis in 1960, as this is the earliest year for which the 1990 occupational cross walk is available. Appendix Table A2 reports the 67 occupations we analyze in our main specification using the 1990 occupational sub-headings. Example occupations include “Executives, Administrators, and Managers”,

\textsuperscript{7}We think an interesting extension would be to include Hispanics in the analysis. In 1960 and 1970, however, there are not enough Hispanics in the data to provide reliable estimates of occupational sorting. Such an analysis can be performed starting in 1980. We leave such an extension to future work.

\textsuperscript{8}For all analysis in the paper, we weight our data using the sample weights available in the different surveys.

\textsuperscript{9}http://usa.ipums.org/usa/volii/99occup.shtml.
“Engineers”, “Natural Scientists”, “Health Diagnostics”, “Health Assessment”, and “Lawyers and Judges”. Appendix Table A3 gives a more detailed description of some of these occupational categories. For example, the “Health Diagnostics” occupation includes physicians, dentists, veterinarians, optometrists, and podiatrists, and the “Health Assessment and Treating” occupations include registered nurses, pharmacists, and dieticians. For short hand, we sometimes refer to these occupations as doctors and nurses, respectively. The way the occupations are defined ensures that each of our occupational categories has positive mass in all years or our analysis.

As seen with the examples above, there is some heterogeneity within our 67 base occupational categories. To assess the importance of such heterogeneity, we perform a series of robustness exercises for many of our main empirical results where we use different levels of occupational aggregation. Specifically, in some robustness specifications, we use the roughly 340 occupations that are consistently defined (using the 1990 occupation codes) in 1980, 1990, 2000, and 2006-8. The reason we start this in 1980 is that the occupational classification system is roughly similar across the Censuses and ACS starting in 1980. We perform our main analysis using the 340 detailed occupation codes for the 1980–2008 period and show that the quantitative outcomes are very similar to what we get using our 67 base occupation codes for the same period. Additionally, we show that much of our quantitative results can be generated if we use only 20 broad occupation categories as opposed to the roughly 67 occupation codes in our base analysis. The 20 occupation categories we use for this robustness analysis are shown in Appendix Table A4. The 20 broad occupation categories include the same universe of 67 occupations just aggregated to broader categories. As we show throughout, our key empirical results come from the fact that women and blacks in recent periods are sorting with a more equal propensity relative to white men in a handful of high skilled occupations.

Our measures of earnings throughout the paper sum together the individual’s labor, business, and farm income. The earnings measures in the Census are from the prior year. Implicitly we assume that individuals who are working in a given occupation in the survey year also worked in that same occupation during the prior year which corresponds to their income report. When measuring earnings, we only
focus on those individuals who worked at least 48 weeks during the prior year and who had at least 1000 dollars of earnings (in year 2007 dollars). We define the wage rate by dividing individual earnings from the prior year by the product of the weeks worked during the prior year and the reported current usual hours worked. When computing individual wage measures, we further restrict the sample to those individuals that report that they usually work more than 30 hours per week.\textsuperscript{10}

For a few of our empirical results, we need a measure of the average wages in the home sector. We impute average earnings for the home sector by extrapolating the relationship between average education and average earnings for the 66 non-home occupations taking into account group fixed effects. Using this year-specific relationship by group and the actual year-specific average education and group composition of participants in the home sector, we predict the average earnings of participants in the home sector.

### 3.2. Occupational Sorting and Wage Gaps By Group

We begin our analysis by documenting the large amount of convergence in the occupational distribution between white men and the other groups over the last fifty years. To illustrate this fact, we create a simple occupational similarity index, $\Psi_g$, which is defined as:

$$\Psi_g \equiv 1 - \frac{1}{2} \sum_{i=1}^{N} |p_{i,wm} - p_{ig}|$$  \quad (13)

To construct the index, we compute the absolute value of the difference in the propensity for a given group to be in an occupation relative to the propensity that white men are in that occupation. We then sum these differences across all occupations. For ease of interpretation, we normalize the measure so that it runs between zero (no occupational overlap between the two groups) and 1 (complete occupa-

\textsuperscript{10}In some census years, weeks worked during the prior year and usual hours worked are reported as categorical variables. In these instances, we use the midpoint of the range when computing the wage rate. See the full details of our data processing in the detailed online data appendix available on the author’s web sites.
tional overlap between the two groups). When computing $\Psi_g$, we exclude the home sector. However, the broad patterns are very similar — particularly the index for white women — even when the home sector is included.

Panel A of Table 1 shows the measure of $\Psi_g$ for white women, black men, and black women in 1960, 1980, and 2008. We also do the comparison for lower educated individuals (those with a high school degree and less) and higher educated individuals (those with more than a high school degree). Within the educational categories, for example, we compare the occupational distribution of lower educated white women to the occupational distribution of lower educated white men.

A few things are of note from Panel A of Table 1. First, each group experienced substantial occupational convergence relative to white men between 1960 and 2008. Second, the timing of the convergence occurred differentially across the groups. For example, occupational convergence occurred both during the 1960 and 1980 period and the 1980 and 2008 period for white women and black women. For black men, however, the bulk of the convergence occurred prior to 1980. Third, there are differences in the occupational convergence by educational attainment. This is seen particularly for white women. In 1960, there were substantial occupational differences both between high educated white women and high educated white men and between low educated white women and low educated white men. Specifically, low educated white men primarily worked in construction and manufacturing while low educated white women primarily worked as secretaries or in low skilled services like food service. High educated white men in 1960 were spread out across many high skilled occupations while high educated white women primarily worked as teachers and nurses. Between 1960 and 2008, however, the occupational similarity between higher educated white men and women converged dramatically while the occupational similarity between lower educated white men and women barely changed. Today, low skilled women still primarily work in services and office support occupations while low skilled men still primarily work in construction and manufacturing.

One of the strong predictions of our occupational sorting model is that the wage gaps relative to white men should be constant for a given group across all occupa-
Table 1: Occupational Similarity and Conditional Wage Gaps Relative to White Men

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<tbody>
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<td>0.49</td>
<td>0.55</td>
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<td>0.53</td>
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<td>0.41</td>
<td>0.44</td>
<td>0.14</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Panel B: Conditional Log Difference in Wages, Relative to White Men

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<tbody>
<tr>
<td>White Women: All</td>
<td>-0.57</td>
<td>-0.47</td>
<td>-0.26</td>
<td>0.10</td>
<td>0.21</td>
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<tr>
<td>White Women: High Educated</td>
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<td>-0.40</td>
<td>-0.24</td>
<td>0.10</td>
<td>0.16</td>
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<td>White Women: Low Educated</td>
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<td>-0.47</td>
<td>-0.27</td>
<td>0.09</td>
<td>0.20</td>
</tr>
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<td>Black Men: All</td>
<td>-0.38</td>
<td>-0.22</td>
<td>-0.15</td>
<td>0.16</td>
<td>0.07</td>
</tr>
<tr>
<td>Black Men: High Educated</td>
<td>-0.29</td>
<td>-0.16</td>
<td>-0.18</td>
<td>0.13</td>
<td>-0.02</td>
</tr>
<tr>
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<td>-0.12</td>
<td>0.15</td>
<td>0.11</td>
</tr>
<tr>
<td>Black Women: All</td>
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<td>-0.48</td>
<td>-0.31</td>
<td>0.38</td>
<td>0.17</td>
</tr>
<tr>
<td>Black Women: High Educated</td>
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<td>-0.39</td>
<td>-0.31</td>
<td>0.23</td>
<td>0.08</td>
</tr>
<tr>
<td>Black Women: Low Educated</td>
<td>-0.88</td>
<td>-0.48</td>
<td>-0.28</td>
<td>0.40</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Note: Panel A of the table reports our occupational similarity index for white women, black men, and black women relative to white men in 1960, 1980, and 2008. The occupational similarity index runs from zero (no overlap in the occupational distribution relative to white men) and one (complete overlap in the occupational distribution relative to white men). The index is computed separately for higher educated and lower educated individuals of the different groups. Panel B reports the difference in log wages between the groups and white men. The entries come from a regression of log wages on group dummies and controls for potential experience and hours worked per week. The regression only includes a sample of individuals working full time. See the text for additional details.
tions. The reason for this is that an occupation that pays a high wage per unit of ability will attract less talented workers. As discussed above, this type of sorting is what makes the wage gap between two groups in a given occupation a poor measure of any differential frictions or absolute advantage between the two groups in that occupation. There are, however, at least three reasons why the estimated wage gaps between groups will not be equated across all occupations. First, there is likely some measurement error in the occupational wage gap estimates due to small sample sizes for some groups in some occupations. Second, although we expect sorting will help offset the effect of differences in wages per ability on the average wage in an occupation, the exact offset due to sorting is a feature of the extreme value distribution. We would not get the complete offset if ability is not exactly distributed according to an extreme value distribution. Third, we focus on occupational sorting due to heterogeneity in ability, but some of the occupational sorting might be driven by other factors such as heterogeneity in preferences. High wage (per unit of ability) occupations might induce the entry of people with high disutility for an occupation rather than individuals with low ability in the occupation. All three forces will generate variation in wage gaps across occupations.

We realize the model is highly stylized but the prediction with respect to the equivalence of wage gaps across occupations for a given occupation seems born out in the data at least when segmenting individuals by accumulated schooling. Panel B of Table 1 shows the estimated wage gap between white men and, respectively, white women, black men, and black women over time and by educational attainment. To obtain these estimates, we regress log wages of the individual on group dummies, a quadratic in potential experience, a polynomial in usual hours worked and our base specification occupation dummies. This regression is estimated only for those individuals who are currently working more than 30 hours per week and who worked at least 48 weeks during the prior year when earnings were measured. We estimated this regression separately for 1960, 1980, and the pooled 2006-2008 sample. The coefficients on the race-sex dummies are shown in the table and should be interpreted as being log deviations relative to white men. We also estimated the regression separately for individuals with 12 years of less of schooling.
and for individuals with more than 12 years of schooling.

As seen in panel B of Table 1, the wage gap for white women relative to white men is nearly identical by educational attainment in 1960, 1980, and 2008. For example, in 1960, low educated white women earned a wage that was 56 log points lower than low educated white men. The comparable number for high educated white women relative to high educated white men was 50 log points. Between 1960 and 2008, the relative wage of low educated white women narrowed by 29 log points. During the same time period, the relative wage of high educated white women narrowed by 26 log points. Despite the fact that the change in the relative occupational similarity was very different by educational attainment for white women (as seen in Panel A), the change in the relative wage gap was nearly identical by educational attainment for white women. According to our model, changes in the relative \( \tau \)'s for white women in high \( \phi \) occupations would generate exactly this result.

Between 1960 and 1980, black men also had a relative wage gap that evolved nearly identically within a sample of lower educated individuals and a sample of higher educated individuals. After 1980, however, there was little change in relative occupational sorting for either high or low skilled black men and there was no change in the relative wage gap for high skilled black men. The wage gap for low skilled black men, however, continued to narrow after 1980. This may be due to the fact that there was a rapid decline in the labor market participation of low skilled black men during the last thirty years that was not random. As currently formulated, our model would not predict such results. However, as we discuss in Section 5, the change in labor market outcomes for black men between 1980 and 2008 do not affect our estimates of aggregate productivity gains in any way.

A further test of the plausibility of our framework is to examine occupation by occupation whether the change in the wage gap between two groups in that occupation is in any way related to the change in the relative propensities of the two groups to be in the occupation. Our model suggests that the two should be unrelated with no variation in the wage gaps. Figure 1 plots the (log) occupational wage gap for white women in 1980 against \( p_{i,ww}/p_{i,wm} \). The latter variable is the relative propensity of a white woman to work in a particular occupation relative to a white
man. As an example, in 1980, white women were 65 times more likely than white men to work as a secretary, but only 0.14 times as likely to work as a lawyer. Given this enormous variation, the difference in the wage gaps between these two occupations is remarkably small. White women secretaries earned about 33 percent less than white men secretaries in 1980, while the gap was 41 percent less for lawyers. Fitting a regression line through the points in the figure shows that there is no relationship at all between the relative wage gap in the occupation and the propensity for white women to be in the occupation relative to white men in 1980.\footnote{The coefficient on the log $p_{i,ww}/p_{i,wm}$ in a regression of the occupational wage gap on log $p_{i,ww}/p_{i,wm}$ was 0.002 with a standard error of 0.008 and an adjusted R-squared of essentially zero. For interpretation, the standard deviation of the independent variable was 1.96 and the mean of the dependent variable was -0.31. The regression was weighted using the number of individuals in the occupation across all groups.} The patterns in other years and for other groups were quite similar. Notice that, within the model, it is the relative propensity that pins down any potential frictions facing a group (relative to white men) in that occupation.\footnote{In additional work (not shown), we explored the relationship between occupational wage gaps and the average earnings of individuals in those occupations. On average, high income occupations tended to have larger wage gaps. This suggests that the extreme value distribution might not entirely correct for high income occupations. Nonetheless, the magnitude of this correlation was almost always small. For example, in 2006-2008, white working women had about a 3 percentage point larger wage gap relative to white men in response to a one-standard deviation increase in occupational log income. As seen from Table 1, the average wage gap was 26 percentage points.}

Our productivity gains in the subsequent sections are based on the change in the occupational distribution over time. Figure 2 shows that the change in log $p_{i,ww}/p_{i,wm}$ between 1960 and 2008 is also uncorrelated with the change in the wage gap between white women and white men between 1960 and 2008. The relative fraction of white women who are doctors increased by 144 percent between 1960 and 2008. For nurses, in contrast, the relative fraction who are white women decreased by 52 percent. Yet the relative wage gap between white men and white women narrowed by between 20 and 30 log points in both occupations. Again, our model predicts that the change in the wage gaps should be uncorrelated with the change in the occupational sorting. This prediction is born out in the data.
Figure 1: Occupational Wage Gaps for White Women in 1980

Note: The figure shows the relationship between the (log) occupational wage gap for white women compared to white men and the relative propensity to work in the occupation between white women and white men, $p_{i,ww}/p_{i,wm}$.

Figure 2: Change in Occupational Wage Gaps for White Women, 1960–2008

Note: The figure shows the relationship between the change in (log) occupational wage gap for white women compared to white men between 1960 and 2008 and the change in the relative propensity to work in the occupation between white women and white men, $p_{i,ww}/p_{i,wm}$, over the same time period.
3.3. Explaining Occupational Differences Across Groups Over Time

Motivated by our model, we use data on the difference in occupational propensities across groups as well as the average wage gaps to infer a composite measure of the occupation-specific frictions. Specifically, given equations (12) and (11), we can define the composite measure for each group (relative to white men) in each occupation as:

\[ \hat{\tau}_{ig} = \frac{\tau_{ig}}{\tau_{i,wm}} \left( \frac{T_{i,wm}}{T_{i,g}} \right)^{\frac{1}{\theta}} \left( \frac{p_{ig}}{p_{i,wm}} \right)^{-\frac{1}{\eta}} \left( \frac{\text{wage}_{wm}}{\text{wage}_g} \right)^{1-\eta}. \]  

(14)

This equation has the following interpretation. If a group is either underrepresented in an occupation or if it faces a large average wage gap, the right-hand side of this equation will be high. The model can explain this in one of two ways (on the left side): either the group faces a large composite friction, or it has a relatively low mean talent in that occupation (e.g. women in occupations where brute strength is important). We observe the right-hand side of this equation in the data and therefore use it to back out the average relative distortion (or talent), \( \hat{\tau}_{ig} \).

To implement this calculation, we require estimates of \( \theta \) and \( \eta \). The parameter \( \theta \) is a key parameter that governs the dispersion of wages. Given the occupational choice model developed above, one can show that the dispersion of wages across people within an occupation-group obeys a Fréchet distribution with the shape parameter \( \theta(1 - \eta) \): the lower is this shape parameter, the more wage dispersion there is within an occupation. Wage dispersion therefore depends on the dispersion of talent (governed by \( 1/\theta \)) and amplification from accumulating human capital via spending (governed by \( 1/(1 - \eta) \)). In particular, the coefficient of variation of wages within an occupation-group in our model satisfies

\[ \frac{\text{Variance}}{\text{Mean}^2} = \frac{\Gamma\left(1 - \frac{2}{\theta(1-\eta)}\right)}{\left(\Gamma\left(1 - \frac{1}{\theta(1-\eta)}\right)\right)^2} - 1. \]  

(15)

To estimate \( \theta(1 - \eta) \) in a given year, we need to know the dispersion of abil-
ity for a given occupation. As a starting point, we look at wage dispersion within occupation-groups. We take residuals from a cross-sectional regression of log worker wages on 66x4 occupation-group dummies. These span the 66 occupations (excluding Home) and the four groups of white men, white women, black men, and black women. The wage is the hourly wage, and the sample includes both full-time and part-time workers. The dummies should capture the impact of schooling requirements ($\phi_i$ levels) on average wages in an occupation, and the wage gaps created by frictions (the average $\tau_{ig}$ across occupations for each group). We calculate the mean and variance across workers of the exponent of these wage residuals. We then solve equation (15) for the value of $\theta(1 - \eta)$. Sampling error is trivial here because there are 300-400k observations per year for 1960 and 1970 and 2-3 million per year for 1980 onward. The point estimates for $\theta(1 - \eta)$ average 3.12. They drift down over time, from around 3.3 in 1960 to 2.9 in 2006-2008, as one would expect given rising wage inequality.

We are concerned that this way of estimating $\theta$ wrongly attributes all of the dispersion of wages within occupation-groups to comparative advantage. We thus make several adjustments, all of which serve to reduce residual wage dispersion. First, we compress the variance of the residuals by 14% and 4%, respectively, to reflect estimates of transitory wage movements from Guvenen and Kuruscu (2009) and how much wage variation can be explained by AFQT scores from Rodgers and Spriggs (1996). Temporary wage differences across workers are not a source of enduring comparative advantage, and AFQT score is arguably correlated with absolute ability across many occupations. Second, we controlled directly for individual education, hours worked, and potential experience in the Census data. Like AFQT score, a worker’s education might be correlated with absolute advantage across many occupations. Though we examine the hourly wage, there could be compensating differentials associated with the workweek. And experience reflects the life cycle rather than lifelong comparative advantage.

These adjustments cumulatively explain 25% of wage variation within occupation-groups. In wages within an occupation to comparative advantage. Attributing the remaining 75% of wage dispersion to comparative advantage, we arrive at a base-
line value of $\theta (1 - \eta) = 3.44$. When computing our counterfactuals in Section 5, we explore the robustness of results to higher values of $\theta (1 - \eta)$ that attribute only 50%, 25% and 10% of the within occupation-group wage dispersion to comparative advantage.

The parameter $\eta$ denotes the elasticity of human capital with respect to education spending. Related parameters have been discussed in the literature, for example by Manuelli and Seshadri (2005) and Erosa, Koreshkova and Restuccia (2010). In our model, $\eta$ will equal the fraction of output spent on accumulating human capital in equilibrium, separate from time spent accumulating human capital. Absent any solid evidence on this parameter, we set $\eta = 1/4$ in our baseline and explore robustness to $\eta = 0$ and $\eta = 1/2$. In general, this parameter slightly affects the level of the $\tau_{ig}$ parameters, but not much else in the results.

Table 2 reports our estimates of $\hat{\tau}_{ig}$ for white women for a subset of our baseline occupations. The occupations we highlight in Table 2 represent five different types of occupations. First, we highlight the $\hat{\tau}_{ig}$ for the home sector. Second, we highlight high educated occupations for which white women were underrepresented in 1960. These occupations include executives, engineers, doctors, and lawyers. Third, we highlight high educated occupations like nursing and teachers for which white women were overrepresented relative to white men in 1960. Finally, we show low education occupations with many white women in 1960 (e.g., secretaries and waitresses) and low education occupations with relatively few white women in 1960 (e.g., construction, firefighters, and vehicle mechanics).

Many interesting patterns emerge from Table 2. First, consider the results for white women in the “home” occupation in 1960. Despite white women being 7 times as likely to work in the home sector as white men, we estimate $\hat{\tau}_{ig}$ for white women in the home sector to be just below 1 (0.99). This implies that white women in 1960 did not have an absolute advantage over white men in the home sector. Two factors underlie our estimate of $\hat{\tau}_{ig}$ being close to 1 in the home sector for white women. First, we are estimating that white women were choosing the home sector because they were facing disadvantages in other occupations. Those barriers show up in the average wage gap between white women and white men. Given that white
women earned roughly 57 percent less when working than white men, our model predicts that women should be much more likely to work in the home sector relative to white men all else equal. Second, how much more white women should be working in the home sector if the other sectors are less attractive for white women depends on $\theta$. As the skill distribution becomes less dispersed ($\theta$ increasing), frictions in other sectors will push more women into the home sector. The reason for this is that the comparative advantage in a given occupation relative to another occupation gets stronger when $\theta$ is higher. Given our estimate of $\theta$, the observed wage gap between white men and white women, and the relative propensity of each group to be in the home sector, we estimate that $\hat{\tau}_{ig}$ is roughly one for white women in the home sector.

As seen from Table 2, $\hat{\tau}_{ig}$ is also close to 1 for white women in the home sector in all years of our analysis. This suggests that women did not move out of the home sector because they lost any absolute advantage in the home sector. Instead, our results suggest that women moved into market occupations due to declining barriers in the market. Below we will show that changes in the productivity of the home sector relative to the market sector for all groups also contributed to women exiting the home sector over this time period. In order to quantify this effect, we need the general equilibrium analysis formulated in the next section. To preview our results,

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**Table 2: Estimated Barriers ($\hat{\tau}_{ig}$) for White Women**

<table>
<thead>
<tr>
<th>Year</th>
<th>Home</th>
<th>Executives/Managers</th>
<th>Engineers</th>
<th>Doctors</th>
<th>Lawyers</th>
<th>Teachers, Non-Postsecondary</th>
<th>Nurses</th>
<th>Secretaries</th>
<th>Food Prep/Service</th>
<th>Construction</th>
<th>Firefighting</th>
<th>Vehicle Mechanic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>0.99</td>
<td>2.39</td>
<td>4.57</td>
<td>3.05</td>
<td>3.43</td>
<td>1.37</td>
<td>1.10</td>
<td>0.76</td>
<td>1.28</td>
<td>4.91</td>
<td>5.94</td>
<td>5.26</td>
</tr>
<tr>
<td>1970</td>
<td>1.05</td>
<td>2.27</td>
<td>4.20</td>
<td>2.88</td>
<td>3.15</td>
<td>1.43</td>
<td>1.08</td>
<td>0.76</td>
<td>1.25</td>
<td>4.02</td>
<td>5.07</td>
<td>3.97</td>
</tr>
<tr>
<td>1980</td>
<td>1.01</td>
<td>1.83</td>
<td>2.94</td>
<td>2.34</td>
<td>2.19</td>
<td>1.26</td>
<td>0.96</td>
<td>0.57</td>
<td>1.16</td>
<td>3.49</td>
<td>3.86</td>
<td>3.58</td>
</tr>
<tr>
<td>1990</td>
<td>1.04</td>
<td>1.49</td>
<td>2.21</td>
<td>1.82</td>
<td>1.71</td>
<td>1.08</td>
<td>0.96</td>
<td>0.53</td>
<td>1.14</td>
<td>2.93</td>
<td>2.97</td>
<td>2.91</td>
</tr>
<tr>
<td>2000</td>
<td>1.00</td>
<td>1.41</td>
<td>2.05</td>
<td>1.58</td>
<td>1.51</td>
<td>0.96</td>
<td>0.93</td>
<td>0.59</td>
<td>1.13</td>
<td>2.72</td>
<td>2.60</td>
<td>2.72</td>
</tr>
<tr>
<td>2008</td>
<td>0.01</td>
<td>1.35</td>
<td>1.90</td>
<td>1.41</td>
<td>1.38</td>
<td>1.14</td>
<td>0.93</td>
<td>0.59</td>
<td>1.14</td>
<td>2.66</td>
<td>2.45</td>
<td>2.71</td>
</tr>
</tbody>
</table>

Note: Author’s calculations based on equation (14) using Census data and imposing $\theta = 3.44$ and $\eta = 1/4$. 

Women earned roughly 57 percent less when working than white men, our model predicts that women should be much more likely to work in the home sector relative to white men all else equal. Second, how much more white women should be working in the home sector if the other sectors are less attractive for white women depends on $\theta$. As the skill distribution becomes less dispersed ($\theta$ increasing), frictions in other sectors will push more women into the home sector. The reason for this is that the comparative advantage in a given occupation relative to another occupation gets stronger when $\theta$ is higher. Given our estimate of $\theta$, the observed wage gap between white men and white women, and the relative propensity of each group to be in the home sector, we estimate that $\hat{\tau}_{ig}$ is roughly one for white women in the home sector.

As seen from Table 2, $\hat{\tau}_{ig}$ is also close to 1 for white women in the home sector in all years of our analysis. This suggests that women did not move out of the home sector because they lost any absolute advantage in the home sector. Instead, our results suggest that women moved into market occupations due to declining barriers in the market. Below we will show that changes in the productivity of the home sector relative to the market sector for all groups also contributed to women exiting the home sector over this time period. In order to quantify this effect, we need the general equilibrium analysis formulated in the next section. To preview our results,
we find that the changing productivity of the home sector relative to the market sector explains roughly 70 percent of the movement of white women out of the home sector. The remaining 30 percent is due to changes in the $\hat{\tau}_{ig}$ in the market sector.

The remainder of the 1960 results from Table 2 reinforce that the $\hat{\tau}_{ig}$'s for white women changed dramatically over time in certain occupations. For example, our estimates of $\hat{\tau}_{ig}$ for executives, lawyers, doctors, and engineers for white women in 1960 ranged from 2.4 to 4.6. In terms of our sorting model, the low relative participation of white women in these occupations in 1960 causes us to infer high values of $\hat{\tau}_{ig}$. The model is attributing the low propensity for white women to work in these occupations as reflecting barriers (either in the human capital market or the labor market directly) or a lower native ability to work in these occupations. Interestingly, the $\hat{\tau}_{ig}$ for white women teachers is also greater than one in 1960. While white women were 1.7 times more likely than white men to work as teachers, this propensity is more than offset by the overall wage gap in 1960, where women earned about 0.57 times what men earned. If white women were not facing some friction or lower absolute advantage in the teacher occupation, our model predicts there should have been an even higher fraction of white women ending up as teachers in 1960.

Contrast this with secretaries in 1960. A white woman in 1960 was 24 times more likely to work as a secretary than was a white man. The model can only explain this enormous discrepancy by assigning a $\hat{\tau}_{ig}$ of 0.76 for white women secretaries. A $\tau$ below 1 is like a subsidy, so the model says either white women had an absolute advantage relative over white men as secretaries, or there was discrimination against white men secretaries. Also in 1960, white women had very high $\hat{\tau}_{ig}$ values in the construction, firefighting and vehicle mechanic professions.

For executives, lawyers, and doctors, the $\hat{\tau}_{ig}$'s fell from around be around 3.0 to being around 1.4 between 1960 and 2008. School teachers also saw a substantial fall in their average $\hat{\tau}_{ig}$ from 1.37 to a value slightly below 1. While barriers facing white women fell in many skilled professional occupations, the $\hat{\tau}_{ig}$ values did not change much for low skilled occupations. This is particularly true after 1980. For example, the estimated $\hat{\tau}_{ig}$ for white women barely changed (or rose) for secretaries, wait-
Table 3: Estimated Barriers ($\hat{\tau}_{ig}$) for Black Men and Women

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Black Men</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Home</td>
<td>1.11</td>
<td>0.96</td>
<td>0.94</td>
<td>-0.15</td>
<td>-0.02</td>
</tr>
<tr>
<td>Executives/Managers</td>
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</tr>
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<td>2.00</td>
<td>1.42</td>
<td>1.21</td>
<td>-0.58</td>
<td>-0.21</td>
</tr>
<tr>
<td>Doctors</td>
<td>1.95</td>
<td>1.60</td>
<td>1.46</td>
<td>-0.36</td>
<td>-0.14</td>
</tr>
<tr>
<td>Lawyers</td>
<td>1.99</td>
<td>1.66</td>
<td>1.50</td>
<td>-0.33</td>
<td>-0.16</td>
</tr>
<tr>
<td>Teachers, Non-Postsecondary</td>
<td>1.39</td>
<td>1.29</td>
<td>1.12</td>
<td>-0.10</td>
<td>-0.17</td>
</tr>
<tr>
<td>Nurses</td>
<td>1.62</td>
<td>1.32</td>
<td>1.17</td>
<td>-0.31</td>
<td>-0.14</td>
</tr>
<tr>
<td>Food Prep/Service</td>
<td>1.07</td>
<td>1.06</td>
<td>1.04</td>
<td>0.00</td>
<td>-0.03</td>
</tr>
<tr>
<td>Construction</td>
<td>1.46</td>
<td>1.25</td>
<td>1.29</td>
<td>-0.22</td>
<td>0.05</td>
</tr>
</tbody>
</table>

| **Black Women**      |      |      |      |                          |                          |
| Home                 | 1.26 | 1.04 | 1.07 | -0.22                    | 0.03                     |
| Executives/Managers  | 3.73 | 2.06 | 1.51 | -1.67                    | -0.55                    |
| Engineers            | 6.45 | 3.19 | 2.25 | -3.26                    | -0.94                    |
| Doctors              | 4.34 | 2.65 | 1.67 | -1.68                    | -0.98                    |
| Lawyers              | 5.08 | 2.47 | 1.65 | -2.61                    | -0.82                    |
| Teachers, Non-Postsecondary | 1.70 | 1.25 | 1.00 | -0.45                    | -0.25                    |
| Nurses               | 1.48 | 1.00 | 0.86 | -0.47                    | -0.15                    |
| Food Prep/Service    | 1.42 | 1.10 | 1.16 | -0.32                    | 0.05                     |
| Construction         | 6.20 | 3.33 | 3.09 | -2.86                    | -0.25                    |

Note: Author’s calculations based on equation (14) using Census data and baseline parameter values.

resses, construction workers, and vehicle mechanics between 1980 and 2008. Yet, the $\hat{\tau}_{ig}$’s for executives, engineers, doctors, lawyers, teachers, and nurses continued to fall during this time period. These results are consistent with the results above showing that occupational convergence was primarily among high skilled individuals.

The $\hat{\tau}_{ig}$’s for black men and black women — for these same select occupations — are shown in Table 3. A similar overall pattern emerges, with the $\hat{\tau}_{ig}$’s being substantially above 1 in general in 1960, but falling through 2008. Still, they remained above 1 by 2008, especially for the high-skilled occupations, suggesting barriers remain. Unlike for white and black women, almost the entire change in the $\hat{\tau}_{ig}$ for black men occurred prior to 1980.
Table 4: Summary Measures of Frictions ($\hat{\tau}_{ig}$) by Group

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>White Women</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>2.09</td>
<td>2.01</td>
<td>1.71</td>
<td>1.49</td>
<td>1.38</td>
<td>1.33</td>
</tr>
<tr>
<td>Variance of Log</td>
<td>0.31</td>
<td>0.23</td>
<td>0.18</td>
<td>0.12</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td><strong>Black Men</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.50</td>
<td>1.34</td>
<td>1.22</td>
<td>1.18</td>
<td>1.18</td>
<td>1.17</td>
</tr>
<tr>
<td>Variance of Log</td>
<td>0.12</td>
<td>0.14</td>
<td>0.10</td>
<td>0.06</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td><strong>Black Women</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>2.84</td>
<td>2.26</td>
<td>1.76</td>
<td>1.53</td>
<td>1.45</td>
<td>1.43</td>
</tr>
<tr>
<td>Variance of Log</td>
<td>0.43</td>
<td>0.25</td>
<td>0.18</td>
<td>0.13</td>
<td>0.11</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Note: Author’s calculations based on equation (14) using Census data and baseline parameter values. We weight all the data using the share of that occupation in the total wage bill.

Table 4 shows the mean and the variance of the log of $\hat{\tau}_{ig}$ for each group over time. As seen from Table 4, not only did the mean $\hat{\tau}_{ig}$ fall for all groups over time, the variance of the log $\hat{\tau}_{ig}$ also fell sharply.\textsuperscript{13} When computing the productivity gains from changes in the $\tau_{ig}$’s in Section 5, it is the variance of the log of the $\tau$’s that drives misallocation. As seen from Table 4, this statistic has fallen sharply for all groups over time.\textsuperscript{14}

3.4. Summary

In this section, we have empirically explored some of the predictions of our occupational sorting model. First, we have shown that wage gaps within an occupation between groups are unrelated to the relative propensity of the groups to be in an occupation. The relative propensities for a group to be in an occupation is a composite measure of differences faced by the groups in occupational frictions (either $\tau_{ig}^h$ or $\tau_{ig}^w$) or differences in absolute advantage between the groups in the occupation ($T_{ig}$). Second, we compute this composite measure ( $\hat{\tau}_{ig}$) for white women, black

\textsuperscript{13} In 1960, there were a few occupations which were populated by no black women. These occupations included architects, fire fighters, vehicle mechanics, electronic repairers, and forest and logging. For these occupations, our measures of $\hat{\tau}_{ig}$ for black women were set to the highest other estimated $\hat{\tau}_{ig}$ for black women in 1960 (across all the occupations for which they populated). Such a normalization does not affect our quantitative results in Section 5 in any meaningful way.

\textsuperscript{14} When showing the mean and standard deviations of the $\hat{\tau}_{ig}$’s, we weight each occupation by their share of earnings in that occupation out of the total wage bill.
men, and black women over time during the last 50 years. The big declines in the composite occurred primarily in high-skilled occupations for white women. This is reflected in the fact that the occupational similarity between high-skilled white men and high-skilled white women have converged to a much greater extent than between low-skilled white men and low-skilled white women.

4. Closing the Model

In order to evaluate the macroeconomic consequences of the changing allocation of talent, we must aggregate across the different occupations in some way. We choose a relatively natural approach and show that our general results are robust to the way we aggregate.

In particular, assume the $N$ occupations combine in a CES fashion to produce a single aggregate output $Y$ according to

$$Y = \left( \sum_{i=1}^{N} (A_i H_i)^\rho \right)^{1/\rho} \quad (16)$$

where $H_i$ denotes the total efficiency units of labor employed in occupation $i$ and $A_i$ is the exogenously-given productivity of the occupation.

The total efficiency units of labor in each occupation are given by

$$H_i = \sum_{g=1}^{G} q_g \int h_{ijg} \epsilon_{ijg} dj. \quad (17)$$

To understand this equation, start from the right. First, we integrate over all people $j$ in group $g$, adding up their efficiency units, which are the product of their human capital and their idiosyncratic ability. Next, there are $q_g$ people belonging to group $g$. Finally, we add up across all the groups.

That completes the setup of the model. We can now define an equilibrium and then start exploring the model’s aggregate implications.
4.1. Equilibrium

A competitive equilibrium in this economy consists of individual choices \{c, e, s\}, an occupational choice by each person, total efficiency units of labor in each occupation \(H_i\), final output \(Y\), and an efficiency wage \(w_i\) in each occupation such that

1. Given an occupational choice, the occupational wage \(w_i\), and idiosyncratic ability \(\epsilon\) in that occupation, each individual chooses \(c, e, s\) to maximize utility:

\[
U(\tau^w, \tau^h, w, \epsilon) = \max_{c,e,s} (1 - s)c^\beta \text{ s.t. } c = (1 - \tau^w_{ig})w\epsilon h(e, s) - e(1 + \tau^h_{ig}).
\] (18)

2. Each individual chooses the occupation that maximizes his or her utility: \(i^* = \arg\max_i U(\tau^w_{ig}, \tau^h_{ig}, w_i, \epsilon_i)\), taking \(\{\tau^w_{ig}, \tau^h_{ig}, w_i, \epsilon_i\}\) as given.

3. A representative firm chooses labor input in each occupation, \(H_i\), to maximize profits:

\[
\max_{\{H_i\}} \left( \sum_{i=1}^{N} (A_i H_i)^\rho \right)^{1/\rho} - \sum_{i=1}^{N} w_i H_i
\] (19)

4. The occupational wage \(w_i\) clears the labor market for each occupation:

\[
H_i = \sum_{g=1}^{G} q_g \int h_{ijg} \epsilon_{ijg} dj
\] (20)

5. Total output is given by the production function in equation (16).

6. “Revenue” associated with the distortions equals zero for each occupation.

The equations characterizing the general equilibrium are then given in the next result.

**Proposition 4 (Solving the General Equilibrium):** The general equilibrium of the model is \(\{p_{ig}, H_i^{\text{supply}}, H_i^{\text{demand}}, w_i\}\) and \(Y\) such that

1. \(p_{ig}\) satisfies equation (8).
2. $H_i^{\text{supply}}$ aggregates the individual choices:

$$H_i^{\text{supply}} = \gamma \tilde{\eta} w_i^{\theta-1} (1 - s_i)^{(\theta(1-\eta)-1)/\beta} \sum_g q_g (1 + \tau_{ig})^{-\eta} (1 - \tau_{ig})^{-1} m_g^{1-\theta(1-\eta)}$$

(21)

3. $H_i^{\text{demand}}$ satisfies firm profit maximization:

$$H_i^{\text{demand}} = \left( \frac{A_i^o}{w_i} \right)^{1/\rho} Y$$

(22)

4. $w_i$ clears each occupational labor market: $H_i^{\text{supply}} = H_i^{\text{demand}}$.

5. Total output is given by the production function in equation (16).

5. Estimating Productivity Gains from Changing Occupational Sorting

5.1. Parameter Values and Exogenous Variables

The key parameters of the model — assumed to be constant over time — are $\eta$, $\theta$, $\rho$, and $\beta$. We discussed the estimation and assumptions for $\eta$ and $\theta$ above. The parameter $\rho$ governs the elasticity of substitution among our 67 occupations in aggregating up to final output. We have little information on this parameter and choose $\rho = 2/3$ for our baseline value. We explore robustness to a wide range of values for $\rho$.

The parameter $\beta$ is the geometric weight on consumption relative to time in an individual’s utility function (1). As schooling trades off time for consumption, the model implies that wages must increase more steeply with schooling in equilibrium when $\beta$ is lower. We choose $\beta = 0.693$ to match the average Mincerian return to schooling across occupations, which averages 12.7% across the six decades.\footnote{Workers must be more heavily compensated for sacrificing time to schooling the more they care about time relative to consumption. To be specific, the average wage of group $g$ in occupation $i$ is proportional to $(1 - s_i)^{\beta}$. If we take a log linear approximation around average schooling $\bar{s}$, then $\beta$ is inversely related to the Mincerian return to schooling across occupations (call this return $\psi$): $\beta = (\psi(1 - \bar{s}))^{-1}$. We calculate $s$ as years of schooling divided by a pre-work time endowment of 25 years.}
results are essentially invariant to this parameter, as documented later.

As our model is static, we infer exogenous variables separately, decade by decade. In each year, we have $6N$ variables to be determined. For each of the $i = 1, \ldots, N$ occupations these are $A_i$, $\phi_i$, and $\tau_{ig}$, where $g$ stands for white men, white women, black women, or black men. We also allow population shares of each group $q_i$ to vary by year to match the data. Finally, we normalize average ability to be the same in each occupation-group, or $T_{ig} = 1$: differences in mean ability across occupations are isomorphic to differences in the production technology $A_i$. Across groups, we think the natural starting point is no differences in mean ability; this assumption will be relaxed in our robustness checks.

To identify the values of these $6N$ forcing variables in each year, we match the following $6N$ moments in the data, decade by decade (numbers in parentheses denote the number of moments):

(4$N$ − 4) The fraction of people from each group working in each occupation, $p_{ig}$. (Less than 4$N$ moments because the $p_{ig}$ sum to one for each group.)

($N$) The average wage in each occupation.

($N$) Zero total revenue from the discrimination “tax” in each occupation.

(3) Wage gaps between white men and each of our 3 other groups.

(1) Average years of schooling in one occupation.

As discussed above, the $\tau_{ig}$ variables are easy to identify in the data given our setup. But recall that $\tau_{ig} \equiv \frac{(1+\tau^h_{ig})^g}{1-\tau_{ig}}$. From the data we currently have, we cannot separately identify the $\tau^h$ and $\tau^w$ components of $\tau$. That is, we cannot distinguish between barriers to accumulating human capital and labor market barriers. We proceed by considering two polar cases. At one extreme, we assume all of the $\tau^w_{ig}$’s are zero, so that $\tau_{ig}$’s solely reflect $\tau^h$’s. At the other extreme, we set all of the $\tau^h_{ig} = 0$ and find the Mincerian return $\psi$ from a regression of log wages on average occupation schooling, with group dummies as controls. We then set $\beta = 0.693$, the simple average of the implied $\beta$ values across years. This method allows the model to approximate the Mincerian return to schooling across occupations.
and assume the $\tau^w_i$'s are responsible for the $\tau$'s. That is, we assume only human capital barriers (the $\tau^h$ case) or only labor market barriers (the $\tau^w$ case).

The $A_i$ levels and the relative $\phi_i$'s across occupations involve the general equilibrium solution of the model, but the intuition for what pins down their values is clear. We already noted that $A_i$ is observationally equivalent to the mean talent parameter in each occupation $T_i$. The level of $A_i$ therefore pins down the overall fraction of the population that works in each occupation. We also noted above that $\phi_i$ is the key determinant of average wage differences across occupations, so it is this moment in the data that pins down its value.

Recall from equation (10) that wages are increasing in schooling across occupations. From Proposition 1, we know that schooling increases with $\phi_i$. Thus we can infer from wages in each occupation the relative values of $\phi_i$ across occupations. But we cannot pin down the $\phi_i$ levels, as wage levels are also affected by the $A_i$ productivity parameters. Thus we use the final moment – average years of schooling in one occupation – to determine the $\phi_i$ levels. We choose to match schooling in the lowest wage occupation, which is Farm Non-Managers. Calling this the min occupation, we set $\phi_{\text{min}}$ in a given year to match the observed average schooling among Farm Non-Managers in the same year: 

$$\phi_{\text{min}} = \frac{1 - \eta}{\beta (1 - s_{\text{min}})}.$$ 

### 5.2. Productivity Gains

Given our model, parameter values, and the forcing variables we infer from the data, we can now answer one of the key questions of the paper: how much of overall earnings growth between 1960 and 2008 can be explained by the changing $\tau$ frictions?

In answering this question, the first thing to note is that output growth in our model is a weighted average of earnings growth in the market sector and in the home sector. Earnings growth in the market sector can be measured as real earnings growth in the census data. Deflating by the NIPA Personal Consumption Deflator, real earnings in the census data grew by 1.32 percent per year between 1960 and 2008.\(^{16}\) For the home sector, we impute wages from the relationship between aver-

\(^{16}\)This might be lower than standard output growth measures because it is calculated solely from wages; for example, it omits employee benefits.
age education and average earnings across market sectors and from wage gaps by group in market sectors. (See the discussion in section 3.1. for additional details.) Taking a weighted average of the imputed wage in the home sector and the wage in the census data, we estimate that output (as defined by our model) grew by 1.47 percent per year between 1960 and 2008.

How much of this growth is accounted for by changing $\tau$'s, according to our model? We would like to answer this question by holding the $A$'s (productivity parameters by occupation), $\phi$'s (schooling parameters by occupation), and $q$'s (group shares of the working population) constant over time and letting the $\tau$'s change. But at which year's value should we hold the $A$'s, $\phi$'s, and $q$'s constant? We follow the standard approach in macroeconomics and use chaining. That is, we compute growth between 1960 and 1970 allowing the $\tau$'s to change but holding the other parameters at their 1960 values. Then we compute growth between 1960 and 1970 from changing $\tau$'s holding the other parameters at their 1970 values. We take the geometric average of these two estimates of growth from changing $\tau$'s. We do the same for other decadal comparisons (1970 to 1980 and so on) and cumulative the growth to arrive at an estimate for our entire sample from 1960–2008.

The results of this calculation are shown in Table 5. When the frictions are interpreted as occurring in human capital accumulation (the $\tau^h$ case), this calculation indicates that the change in occupational frictions contributed an average of 0.294 percentage points to growth per year. This would explain 20.0 percent of overall earnings growth over the last half century.

If we instead interpret the frictions as occurring in the labor market (the $\tau^w$ case), chain-weighted growth from changing $\tau^w$'s is 0.233 percent per year. According to this case, changing labor market frictions account for 15.8 percent of the cumulative earnings growth from 1960 to 2008. The gains are smaller in the $\tau^w$ case because some of the wage gaps are accounted for directly by labor market discrimination in this case, with no direct implications for productivity. There are still indirect effects operating through human capital accumulation and occupational choice, of course.

A related calculation, perhaps more transparent, is to hold the $\tau$'s constant and
Table 5: Productivity Gains: Share of Growth due to Changing Frictions

<table>
<thead>
<tr>
<th>Frictions in all occupations</th>
<th>$\tau^h$ case</th>
<th>$\tau^w$ case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counterfactual: wage gaps halved</td>
<td>17.0%</td>
<td>13.1%</td>
</tr>
<tr>
<td>Counterfactual: zero wage gaps</td>
<td>12.8%</td>
<td>10.1%</td>
</tr>
<tr>
<td>No frictions in “brawny” occupations</td>
<td>17.8%</td>
<td>13.4%</td>
</tr>
</tbody>
</table>

Note: Average annual wage growth between 1960 and 2008 was 1.47%. Entries in the table show the share of this growth attributable to changing frictions according to our model under various assumptions. In the last line, we assume that there are no frictions for white women in occupations where physical strength is important. Instead, we allow $T_{i,ww}$ to change over time to match the occupational allocation for white women. For blacks in this case, we do allow for frictions, but also assume $T_{i,bw} = T_{i,ww}$.

calculate the hypothetical growth rate due to the changes in the $A$’s, $\phi$’s, and $q$’s. Figure 3 plots the results of this calculation. The left panel considers the $\tau^h$ case, while the right panel corresponds to the $\tau^w$ case. The large majority of wage growth is due to increases in $A_i$ and $\phi_i$ over time, but an important part is attributable to reduced frictions. Allowing the $\tau^h$’s to change as they did historically raises output by 14.8 percent in the $\tau^h$ case.

The right panel of Figure 3 presents similar estimates, this time for the $\tau^w$ case. Here, reduced frictions raised overall output by 11.6% between 1960 and 2008. Note that output growth would have been negative in the 1970s in the absence of the reduction in $\tau$’s for blacks and women in that decade. But over the entire sample, the bulk of growth is due to rising $A$’s and $\phi$’s.

It is worth elaborating on the gains from the changing $\tau$’s. To this end, Figure 4 presents the mean and variance of $\tau^h$ over time for each group in the $\tau^h$ case. The left panel shows the average $\tau$’s falling for women and African-Americans, whereas the barriers facing white males are basically zero throughout. According to the model, these average $\tau$’s led blacks and women to underinvest in human capital (presuming $\eta > 0$). Over time these gaps in average $\tau$’s diminished, leading to a bet-
Figure 3: Counterfactuals: Output Growth due to $A$, $\phi$ versus $\tau$

Note: These graphs show the counterfactual path of output in the model if the $\tau$’s were kept constant over time (in a chained calculation). That is, how much of cumulative growth is due to changing $A$’s and $\phi$’s versus changing $\tau$’s. The left panel is for the $\tau^h$ calibration, and the right is for the $\tau^w$ calibration.

The right panel of Figure 4 shows that the $\tau$’s were also more dispersed across occupations for blacks and (especially) women than for white men. It is this dispersion that leads to misallocation of talent across occupations. If there were no dispersion in the $\tau$’s across occupations for each group, there would be no misallocation of talent. All groups would have the same occupational distributions. The dispersion in the $\tau$’s leads to different occupational choices for each group – which indicates misallocation if the distribution of talent is the same in each group. The falling variance of the log $\tau$’s leads to a better allocation of talent and hence some productivity growth.\footnote{Even for women, most of the decline in the variance can be seen between market occupations. But a notable portion of the overall decline for women comes from falling barriers in market occupations relative to the home sector.}

Could the productivity gains we estimate be inferred from a back-of-the-envelope calculation involving the wage gaps alone? In particular, suppose one takes white male wage growth as fixed, and calculates how much of overall wage growth comes from the faster growth of wages for the other groups. The answer is that faster wage growth for blacks and white women contributed 0.18 percentage points per year to overall wage growth from 1960 to 2008. This is compared to our estimate of the pro-
ductivity gains from changing \( \tau \)'s of 0.30 percent per year in the \( \tau^h \) case and 0.23 percent per year in the \( \tau^w \) case.

Our counterfactual calculations differs from this back-of-the-envelope calculation in two fundamental ways. First, we are isolating the contribution of changing \( \tau \)'s, whereas the back-of-the-envelope also reflects any impact of changing \( A \)'s, \( \phi \)'s, and \( q \)'s on the wage growth of women and blacks relative to men. Second, our counterfactuals take into account the impact on white male wage growth, which is implicitly assumed to be zero in the counterfactual. In our counterfactuals, the wage gains to women and blacks come (partly) at the expense of white males. Recall that we normalize the mean \( \tau \) across groups to zero in each occupation. Thus falling \( \tau \)'s for women and blacks (from positive to smaller positive values) go hand in hand with rising \( \tau \)'s for white men (from negative to less negative values). This is a direct force lowering wages of white men as barriers to blacks and women fall. There are also GE forces operating through the wages per unit of human capital in occupations dominated by white men vs. other groups.

The middle rows of Table 5 quantify the point that changes in the average wage gaps among groups (within occupations) are not primarily responsible for the gains that we estimate. In particular, we consider counterfactuals in which we substan-
tially reduce the average wage gaps in the data that are used in our calibration. Cutting the wage gaps in half only reduces the share of growth explained slightly, for example from 20.0% to 17.0% in the \( \tau^h \) case. Even setting the average wage gaps in the data to zero still leaves 12.8% of growth explained by changes in the human capital frictions. The upshot is that model productivity gains cannot be gleaned from the wage gaps alone.

The final row in Table 5 considers the robustness of our productivity gains to relaxing the assumption that men and women draw from the same distribution of talent in all occupations. In particular, we consider the possibility that some occupations rely more on physical strength than others, and that this reliance might have changed because of technological progress. For this check, we go to the extreme in assuming that frictions for white women are completely absent from the set of occupations where physical strength is arguably important, including firefighters, police officers, and most of manufacturing. That is, we estimate values for \( T_{i,g} \) for white women that completely explain their observed allocation to these occupations for every period between 1960 and 2008. Our hypothesis going into this check was that most of the productivity gains were coming from the rising propensity for women to enter occupations like lawyers, doctors, scientists, professors, and managers, where physical strength is not important. Indeed, the results in Table 5 support this hypothesis. The amount of wage growth explained by changing frictions falls only slightly — for example, from 20.0% to 17.8% in the \( \tau^h \) calibration.

How much additional growth could be had from reducing the frictions all the way to zero? The answer is shown in Table 6. Consider first the \( \tau^h \) calibration. Between 1960 and 2008, changing frictions raised output by 14.8%, as discussed above. If the frictions as of 2008 were removed entirely, output would be higher by an additional 9.3%. For the \( \tau^w \) calibration and when brawny occupations are explained entirely by talent differences, the corresponding numbers are somewhat lower.

---

18 These occupations are assigned based on Rendall (2010).
Table 6: Potential Remaining Output Gains from Zero Barriers

<table>
<thead>
<tr>
<th></th>
<th>$\tau^h$ case</th>
<th>$\tau^w$ case</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Frictions in all occupations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cumulative gain, 1960–2008</td>
<td>14.8%</td>
<td>11.6%</td>
</tr>
<tr>
<td>Remaining gain from zero barriers</td>
<td>9.3%</td>
<td>4.3%</td>
</tr>
<tr>
<td><strong>No frictions in “brawny” occupations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cumulative gain, 1960–2008</td>
<td>13.1%</td>
<td>9.7%</td>
</tr>
<tr>
<td>Remaining gain from zero barriers</td>
<td>7.2%</td>
<td>3.0%</td>
</tr>
</tbody>
</table>

5.3. Robustness

How sensitive is the growth contribution of changing $\tau$’s to our chosen parameter choices? Tables 7 and 8 explore robustness to different parameter values. For each alternative set of parameter values, we recalculate the $\tau_{ig}$, $A_i$, and $\phi_i$ values so that the model continues to fit the occupation shares and wage gaps.

The first row checks sensitivity to the elasticity of substitution ($\rho$) between occupations in production. Under the $\tau^h$ case, the share of growth explained ranges from 15.9 percent when the occupations are almost Leontief ($\rho = -90$) to 23.1 percent when they are almost perfect substitutes ($\rho = 0.95$). This compares to 20.0 percent with our baseline value of $\rho = 2/3$. Outcomes are more sensitive under the $\tau^w$ case, with the share of growth explained by changing $\tau^w$’s going from 10.0 to 19.9 percent (vs. 15.8 percent baseline). Note that gains are increasing in substitutability. Our intuition is that distortions to the total amount of human capital in one occupation vs. another are greater with higher substitutability across occupations (higher $\rho$). We not only have too few women doctors, for example, but too little total human capital of doctors when women face barriers to the medical profession (as men are subsidized, but by a lesser extent than women are discouraged in order for
Table 7: Robustness Results: Percent of Growth Explained in the $\tau^h$ case

<table>
<thead>
<tr>
<th>Changing $\rho$</th>
<th>Baseline</th>
<th>$\rho = 2/3$</th>
<th>$\rho = -90$</th>
<th>$\rho = -1$</th>
<th>$\rho = 1/3$</th>
<th>$\rho = .95$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Changing $\theta(1 - \eta)$</td>
<td>3.44</td>
<td>4.16</td>
<td>5.61</td>
<td>8.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Changing $\eta$</td>
<td>20.0%</td>
<td>18.8%</td>
<td>17.8%</td>
<td>17.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Changing $\beta$</td>
<td>20.0%</td>
<td>20.0%</td>
<td>20.0%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Entries in the table represent the share of earnings growth that is explained by the changing $\tau^h$'s using the chaining approach. Each entry changes one of the parameter values relative to our baseline case.

The second row indicates that the gains from changing $\tau$'s shrink as $\theta(1 - \eta)$ rises above our baseline value (holding $\eta$ fixed at 0.25). Recall that our baseline $\theta(1 - \eta)$ of 3.44 was estimated from wage dispersion within occupation-groups controlling for hours worked, potential experience, and education – and making adjustments for AFQT scores and transitory wage movements. This baseline value attributes 75% of wage dispersion within occupation-groups to comparative advantage. This value may overstate the degree of comparative advantage, as it imperfectly controls for absolute advantage. We thus entertain higher values of $\theta$ that attribute 50

The direction of these results is intuitive: when people are more similar in ability ($\theta$ is higher), smaller barriers are required to explain why women and blacks were underrepresented in high skill occupations. As the more modest barriers drop, the
women and blacks who enter high skill professions are similar in ability to the white men they displace, limiting the gains from reallocation.

The third row considers different values of the elasticity of human capital with respect to goods invested in human capital ($\eta$). The gains are generally increasing in $\eta$. In the $\tau^h$ case, the gains rise from 14.3 percent to 20.0 percent as $\eta$ rises from 0.005 to 0.25.\footnote{We must have $\eta > 0$ in the $\tau^h$ case as the only source of wage and occupation differences across groups is different human capital investments in this case.} Gains are much less sensitive to $\eta$ in the $\tau^w$ case, rising from 15.6 percent to 15.8 percent as $\eta$ goes from 0 to 0.5.

The final row of the robustness Tables shows that the results are not at all sensitive to the weight placed on time vs. goods in utility ($\beta$).
Table 9: Female Participation Rates

<table>
<thead>
<tr>
<th></th>
<th>$\tau^h$ calibration</th>
<th>$\tau^w$ calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Women's LF participation</strong></td>
<td>1960 = 0.329</td>
<td>2008 = 0.692</td>
</tr>
<tr>
<td><strong>Change, 1960 – 2008</strong></td>
<td></td>
<td>0.364</td>
</tr>
<tr>
<td>Due to changing $\tau$'s</td>
<td>0.106</td>
<td>0.116</td>
</tr>
<tr>
<td>(Percent of total)</td>
<td>(29.1%)</td>
<td>(31.8%)</td>
</tr>
</tbody>
</table>

Note: Results are for white women and black women combined. Participation is defined as working in market occupations. The sampling weight of part-time workers is split evenly between the market sector and the home sector. Italicized entries in the table are data; non-italicized entries are results from the model.

5.4. Further Results

Here we describe a number of additional insights from the model.

In the Census data, the share of women working in the market rose from 32.9 percent in 1960 to 69.2 percent in 2008. One explanation is that women’s market opportunities rose, say due to declining discrimination or better information. See Jones, Manuelli, and McGrattan (2003), Albanesi and Olivetti (2009), and Fogli and Veldkamp (2011) for empirical analysis of these hypotheses. As Table 2 showed, the $\tau$’s fell in market occupations relative to the home sector for women. How much of the rising female labor-force participation rate can be traced to changing $\tau$’s? Table 9 provides the answer. Of the 36.3 percentage point increase, the changing $\tau$’s contributed 14.6 percentage points, or around 40 percent of the total increase. According to our model, the remaining 60 percent can be attributed to changes in technology such as the $A$’s. This is in the spirit of the work by Greenwood, Seshadri and Yorukoglu (2005) on “engines of liberation”.

As we report in Table 10, gaps in average years of schooling narrowed from 1960 to 2008 for all three groups vs. white males: by 0.4 years for white women, 1.8 years
for black men, and 1.55 years for black women. If the $\tau$’s for blacks and women fell faster in occupations with above-average schooling, then the changing $\tau$’s contributed to this educational convergence. The Table indicates how much. For white women, the changing $\tau$’s account for the trend and then some (0.6 years, vs. 0.4 in the data). For black men, falling frictions might have narrowed the schooling gap by a similar 0.6 years, but this is only about one-third of the convergence in the data. For black women, declining distortions might explain 70 percent (1.10/1.55) of their catch-up in schooling.

How much of the productivity gains reflect changes in the occupational frictions facing women vs. those facing blacks? Tables 11 and 12 provide the answer for $\tau^h$ and $\tau^w$, respectively. The second column presents the overall wage growth for each time period. The third column replicates the estimates (already shown in Figures 3) of setting the $\tau$’s to their levels at the end of each period (1960–1980, 1980–2008, and 1960–2008 for Rows 1, 2, and 3). Take the $\tau^h$ case. Almost two-thirds (13.0 out of 20.6) of the total gains from reduced occupational frictions over the last fifty years can be explained by the changes facing white women. Falling frictions faced by blacks accounted for two-fifths of the gains.

The share of gains associated with falling frictions for white women vs. blacks
Table 11: Contribution of Each Group to Total Earnings Growth, $\tau^h$ case

<table>
<thead>
<tr>
<th>Year</th>
<th>Base Model Growth</th>
<th>Setting All $\tau^h$'s to End Levels</th>
<th>Setting WM $\tau^h$'s to End Levels</th>
<th>Setting WW $\tau^h$'s to End Levels</th>
<th>Setting BM $\tau^h$'s to End Levels</th>
<th>Setting BW $\tau^h$'s to End Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960-1980</td>
<td>33.3 percent</td>
<td>21.1%</td>
<td>4.2%</td>
<td>8.7%</td>
<td>3.4%</td>
<td>4.7%</td>
</tr>
<tr>
<td>1980-2008</td>
<td>49.8 percent</td>
<td>19.2%</td>
<td>1.5%</td>
<td>15.4%</td>
<td>0.8%</td>
<td>1.4%</td>
</tr>
<tr>
<td>1960-2008</td>
<td>99.7 percent</td>
<td>20.0%</td>
<td>2.6%</td>
<td>12.6%</td>
<td>1.9%</td>
<td>2.8%</td>
</tr>
</tbody>
</table>

Note: Author’s calculations using Census data and baseline parameter values.

differs across the time periods. Again, consider the $\tau^h$ case. Blacks accounted for a larger share of the gains in the 1960s and 1970s than in later decades. From 1960 to 1980, reduced frictions for blacks account for a quarter of the overall gains from reduced frictions. From 1980 to 2008, reduced frictions for blacks account for less than one-tenth of the overall gains. This timing might link the gains for blacks to the Civil Rights movement of the 1960s.

What was the consequence of shifting occupational frictions for the wage growth of different groups? Tables 13 try to answer this question. The first column presents the actual growth of real wages for the different groups from 1960 to 2008. Real wages increased by 77 percent for white men, 126 percent for white women, and 143 percent for both black men and black women. For brevity, consider the $\tau^h$ case. In the absence of the change in occupational frictions, the model says real wages for white men would have been 3 percent higher. Put differently, real income of white men declined due to the changing opportunities for blacks and women. But at the aggregate level, this loss was swamped by the wage gains for blacks and women.
Almost 40 percent of the wage growth for white women was due to the change in occupational frictions. For blacks, around half of their earnings growth might be attributable to the increased opportunities they faced. The model explains the remainder of growth as resulting from changes in technology ($A$'s) and skill requirements ($\phi$'s).

Tables 14 and 15 look at the regional dimension of the decline in frictions confronting blacks and women. Here, we assume that workers are immobile across regions. With this assumption, a decline in occupational frictions in the South relative to the North will increase average wages in the South relative to the North. From 1960 to 2008, wages in the South increased by 10 percent relative to wages in the Northeast. In the $\tau^h$ case, about 5 percentage points of this convergence was due to reduced occupational frictions facing blacks and women in the South relative to the Northeast — with the bulk of the effect due to falling $\tau$'s for blacks.

From 1980 to 2008, we see a reversal of the North-South convergence, perhaps driven by the reverse migration of blacks to the U.S. South. Reverse migration is

**Table 12: Contribution of Each Group to Total Earnings Growth, $\tau^w$ case**

<table>
<thead>
<tr>
<th>Year</th>
<th>Base Model Growth</th>
<th>Setting All $\tau^w$'s to End Levels</th>
<th>Setting WM $\tau^w$'s to End Level</th>
<th>Setting WW $\tau^w$'s to End Level</th>
<th>Setting BM $\tau^w$'s to End Level</th>
<th>Setting BW $\tau^w$'s to End Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960-1980</td>
<td>33.3 percent</td>
<td>21.3%</td>
<td>1.8%</td>
<td>13.8%</td>
<td>2.1%</td>
<td>3.6%</td>
</tr>
<tr>
<td>1980-2008</td>
<td>49.8 percent</td>
<td>11.9%</td>
<td>-0.7%</td>
<td>10.7%</td>
<td>0.4%</td>
<td>1.4%</td>
</tr>
<tr>
<td>1960-2008</td>
<td>99.7 percent</td>
<td>15.8%</td>
<td>0.3%</td>
<td>12.0%</td>
<td>1.1%</td>
<td>2.3%</td>
</tr>
</tbody>
</table>

Note: Author’s calculations using Census data and baseline parameter values.
what one would expect to see if workers are responding to the improved labor market outcomes in the South by relocating to the South. In a long run with higher labor mobility, the main effect of declining occupational frictions for blacks in the South relative to the North might be to increase the number of blacks living in the South relative to the North. Persistent wage gaps might reflect skill differences between regions. Of course, to the extent mobility is costly even in the long run, frictions can contribute to wage gap differences across regions even in the long run.

### 5.5. Average Quality of Workers by Occupation

Using equations (9) and (10), the average quality of workers — including both innate ability and human capital — for group $g$ in occupation $i$ is given by

$$
\frac{H_{ig}}{q_{ig}p_{ig}} = \frac{\gamma \bar{\eta} \cdot \frac{1}{(1 - \tau_{ig}^w)w_i} \cdot (1 - s_i)^{-1/\beta} \cdot \left( \sum_{n=1}^{N} \bar{w}_{ig}^{\bar{n}} w_{ig}^{s} \right)^{\frac{1}{\beta} \frac{1}{1 - \eta}}}{(1 - \tau_{i,wm})w_{i,wm}}.
$$

That is, relative quality in an occupation is simply the wage gap adjusted by the $\tau^w$ frictions.
Table 14: Contributions to Northeast - South Convergence, $\tau^h$ case

<table>
<thead>
<tr>
<th>Year</th>
<th>Base Model Convergence in P. Points</th>
<th>Setting All $\tau^h$'s to End Levels</th>
<th>Setting BM and BW $\tau^h$'s to End Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960-1980</td>
<td>20.7</td>
<td>2.9</td>
<td>3.3</td>
</tr>
<tr>
<td>1980-2008</td>
<td>-16.5</td>
<td>0.2</td>
<td>1.6</td>
</tr>
<tr>
<td>1960-2008</td>
<td>10.0</td>
<td>3.3</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Note: Author's calculations using Census data and baseline parameter values.

Table 15: Contributions to Northeast - South Convergence, $\tau^w$ case

<table>
<thead>
<tr>
<th>Year</th>
<th>Base Model Convergence in P. Points</th>
<th>Setting All $\tau^w$'s to End Levels</th>
<th>Setting BM and BW $\tau^w$'s to End Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960-1980</td>
<td>20.7</td>
<td>1.9</td>
<td>2.2</td>
</tr>
<tr>
<td>1980-2008</td>
<td>-16.5</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>1960-2008</td>
<td>10.0</td>
<td>2.1</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Note: Author's calculations using Census data and baseline parameter values.
In the $\tau^h$ case (where the $\tau^w$ variables are set to zero), equation (24) implies
that average quality for a group relative to white men is *the same across all occup-
pations*. In particular, relative quality is precisely equal to the wage gap. When the
labor market friction are introduced, this changes. In this case, wages are not equal
to marginal products, so that average quality differs from wage. More specifically,
wages are less than marginal products, so average quality is larger when the frictions
are larger.

One way to think about these quality differences is to consider the following
question: if you were to see a doctor chosen at random in 1960, would you rather
see a male doctor or a female doctor?

Figure 5 shows the ratio of average quality between white women and wh ite men
for several occupations, as in equation (24). In the $\tau^h$ case, as noted above, relative
qualities are equated for all occupations. Because the wage gap reflects quality, the
average female doctor is less qualified than the average male doctor — the distor-
tions lead women doctors to accumulate less human capital th an their male coun-
terparts. As the wage gaps have declined over time, the relative quality of women
to men in each occupation rose substantially between 1960 and 2008, from 0.56 to
0.77.

The $\tau^w$ case presents a very different view of the data, as shown in the right
panel. The relative quality of women is higher than their wage gaps suggest be-
cause they are paid less than their marginal products. In 1960, average was sub-
stantially higher for women relative to men for doctors and managers. Only the
most talented women overcame frictions to become doctors and managers in 1960,
and some lesser talented white men entered these professions instead. According to
this case, the difference in quality has faded substantially over time due to declining
frictions, but remains present even in 2008.

Of course, the real world likely reflect forces from both the $\tau^h$ and the $\tau^w$ cases.
To this end, independent information on quality trends for occupation-groups could
be quite helpful in quantifying the relative contributions of human capital and labor
market frictions.
6. Conclusion

How does discrimination in labor markets and in the acquisition of human capital affect occupational choice? And what are the consequences of the resulting misallocation of talent on aggregate productivity? We develop a framework to tackle these questions empirically. This framework has three building blocks. First, we use a standard Roy model of occupational choice, augmented to allow for labor market discrimination and discrimination in the acquisition of human capital. Second, we impose the assumption that the distribution of an individual’s ability over all possible occupations follows an iid extreme value distribution. Third, we embed the Roy model in general equilibrium to account for the effect of occupational choice on the price of skills in each occupation, and to allow for the effect of technological change on occupational choice.

We apply this framework to measure the changes in barriers to occupational choice facing women and blacks in the U.S. from 1960 to 2008. We find large reductions in these barriers, largely concentrated in high-skilled occupations. We then use our general equilibrium setup to measure the aggregate effects of the reduction in occupational barriers facing these groups, observed between 1960 and 2008.
estimate that the reduction in these barriers can explain 15 to 20 percent of aggregate wage growth, 90 to 95 percent of the wage convergence between women and blacks and white men, and 30 percent of the rise in women's labor force participation from 1960 to 2008.

It should be clear that this paper only provides a preliminary answer to these important questions. It would be useful to develop a framework that does not rely on the assumption that the distribution of talent is iid across all occupations (we are less troubled by the extreme value distribution assumption). For example, Lagakos and Waugh (2011) allow skill to be correlated for working inside and outside of agriculture. Mulligan and Rubinstein (2008) do the same for working in the home sector and in the market, and argue for changing selection of high ability U.S. women into the market over time. It would also be useful to quantify the extent to which the barriers are due to labor market discrimination versus discrimination in the acquisition of human capital. As we've discussed, independent data on quality trends would be useful to distinguish between these two forces. Finally, we have focused on the gains from the reduction in barriers in occupational choice facing women and blacks over the last fifty years. However, we suspect that similar barriers facing children from less affluent families and from regions of the country hit by adverse economic shocks have worsened in the last few decades. If so, this could explain both the adverse trends in aggregate productivity and the fortunes of less-skilled Americans over the last decades. We hope to tackle some of these questions in future work. One could also investigate groups in other countries. For example, Hnatkovska, Lahiri and Paul (2011) look at castes in India and find narrowing differences in education, occupations, and wages in recent decades.

A Derivations and Proofs

The propositions in the paper summarize the key results from the model. This appendix shows how to derive the results.

Proof of Proposition 1. Occupational Choice
As given in equation (5), the individual’s utility from choosing a particular occupation is

\[ U(\tau_i, w_i, \epsilon_i) = (\bar{\eta} \bar{w}_{ig} \epsilon_i)^{\frac{\theta}{1-\theta}}, \]

where \( \bar{w}_{ig} \equiv w_i s_i^{\phi_i} (1 - s_i)^{\frac{1}{\tau_i g}} \). The solution to the individual’s problem, then, involves picking the occupation with the largest value of \( \bar{w}_{ig} \epsilon_i \). To keep the notation simple, we will suppress the \( g \) subscript in what follows.

Let \( p_i \) denote the probability that the individual chooses occupation \( i \). Then

\[ p_i = \Pr[\bar{w}_i \epsilon_i > \bar{w}_s \epsilon_s] \lor i \neq s \]
\[ = \Pr[\epsilon_s < \bar{w}_i \epsilon_i / \bar{w}_s] \lor s \neq i \]
\[ = \prod_{s \neq i} F_s(\bar{w}_i \epsilon_i / \bar{w}_s) \]  

(25)

if \( \epsilon_i \) is known for certain. Since it is not, we must also integrate over the probability distribution for \( \epsilon_i \):

\[ p_i = \int \prod_{s \neq i} F_s(\bar{w}_i \epsilon_i / \bar{w}_s) f_i(\epsilon_i) d\epsilon_i, \]  

(26)

where \( f_i(\epsilon) = \theta T_i \epsilon^{-(1+\theta)} \exp\{-T_i \epsilon^{-\theta}\} \) is the pdf of the Fréchet distribution. Substituting in for the distribution and pdf, additional algebra leads to

\[ p_i = \int \theta T_i \left( \prod_{s \neq i} \exp\{-T_s(\bar{w}_i \epsilon_i / \bar{w}_s)^{-\theta}\} \right) \epsilon_i^{-(1+\theta)} \exp\{-T_i \epsilon^{-\theta}\} d\epsilon_i \]
\[ = \int \theta T_i \epsilon_i^{-(1+\theta)} \exp\left\{- \sum_{s=1}^{N} T_s \left( \frac{\bar{w}_i}{\bar{w}_s} \right)^{-\theta} \epsilon_i^{-\theta} \right\} d\epsilon_i. \]  

(27)
Now, define $T_i \equiv - \sum_{s=1}^{N} T_s \left( \frac{\theta_i}{\tilde{w}_i} \right)^{-\theta}$. Then the probability simplifies considerably:

$$p_i = \frac{T_i}{\tilde{T}_i} \int \theta \tilde{T}_i \epsilon_i^{-(1+\theta)} \exp\{-\tilde{T}_i \epsilon_i^{-\theta}\} d\epsilon_i = \frac{T_i}{\tilde{T}_i} \int d\tilde{F}_i(\epsilon_i)$$

$$= \frac{T_i \tilde{w}_i^{\theta}}{\sum_s T_s \tilde{w}_s^{\theta}} = \frac{\tilde{w}_i^{\theta}}{\sum_s \tilde{w}_s^{\theta}} \quad (28)$$

where $\tilde{w}_i \equiv T_i^{1/\theta} \tilde{w}_i$.

Total efficiency units of labor supplied to occupation $i$ by group $g$ are

$$H_{ig} = q_g p_{ig} \cdot \mathbb{E} [h_i \epsilon_i | \text{Person chooses } i].$$

Recall that $h(e, s) = s^\phi e^\eta$. Using the results from the individual's optimization problem, it is straightforward to show that

$$h_i \epsilon_i = \tilde{h}_i (w_i / \tau_i)^{1-\eta} \epsilon_i^{1/\eta},$$

where $\tilde{h}_i \equiv \tilde{\eta} s_i^{1-\eta}$ and $\tilde{\eta} \equiv \eta^{\eta/(1-\eta)}$. Therefore,

$$H_{ig} = q_g p_{ig} \tilde{h}_i (w_i / \tau_i)^{\eta/(1-\eta)} \cdot \mathbb{E} \left[ \epsilon_i^{1/\eta} | \text{Person chooses } i \right]. \quad (29)$$

To calculate this last conditional expectation, we use the extreme value magic of the Fréchet distribution. Let $y_i \equiv \tilde{w}_i \epsilon_i$ denote the key occupational choice term. Then

$$y^* \equiv \max_i \{ y_i \} = \max_i \{ \epsilon_i / \tau_i \} = \epsilon^*/\tau^*.$$
Since \( y_i \) is the thing we are maximizing, it inherits the extreme value distribution:

\[
\Pr [ y^* < z ] = \prod_{i=1}^N \Pr [ y_i < z ] \\
= \prod_{i=1}^N \Pr [ \bar{w}_i \epsilon_i < z ] \\
= \prod_{i=1}^N \Pr [ \epsilon_i < z / \bar{w}_i ] \\
= \prod_{i=1}^N \exp \left\{ -T_i \left( \frac{z}{\bar{w}_i} \right)^{-\theta} \right\} \\
= \exp \left\{ -\sum_{i=1}^N T_i \bar{w}_i^\theta \cdot z^{-\theta} \right\} \\
= \exp \left\{ -Tz^{-\theta} \right\} .
\] (30)

That is, the extreme value also has a Fréchet distribution, with a mean-shift parameter given by \( \bar{T} \equiv \sum_i T_i \bar{w}_i^\theta \).

Straightforward algebra then reveals that the distribution of \( \epsilon^* \), the ability of people in their chosen occupation, is also Fréchet:

\[
G(x) \equiv \Pr [ \epsilon^* < x ] = \exp \left\{ -T^* x^{-\theta} \right\}
\] (31)

where \( T^* \equiv \sum_{i=1}^N T_i (\bar{w}_i / \bar{w}^*)^\theta \).

Finally, one can then calculate the statistic we needed above back in equation (29): the expected value of the chosen occupation's ability raised to some power. In particular, let \( i \) denote the occupation that the individual chooses, and let \( \alpha \) be some positive exponent. Then,

\[
\mathbb{E} [ \epsilon_i^\lambda ] = \int_0^\infty \epsilon^\lambda dG(\epsilon) \\
= \int_0^\infty \theta T^* e^{-(1+\theta)+\lambda} e^{-T^* e^{-\theta}} d\epsilon
\] (32)

Recall that the “Gamma function” is \( \Gamma(\alpha) \equiv \int_0^\infty x^{\alpha-1} e^{-x} dx \). Using the change-of-
variable \( x = T^* \epsilon^{-\theta} \), one can show that

\[
E[\epsilon_i^\lambda] = T^{\lambda/\theta} \int_0^\infty x^{-\lambda/\theta} e^{-x} \, dx \\
= T^{\lambda/\theta} \Gamma(1 - \lambda/\theta).
\] (33)

Applying this result to our model, we have

\[
E \left[ \epsilon_i^{1-\eta} \mid \text{Person chooses } i \right] = T^{1/\theta} \frac{1}{1-\eta} \Gamma \left( 1 - \frac{1}{\theta} \cdot \frac{1}{1-\eta} \right) \\
= \left( \frac{T_i}{p_{i|g}} \right)^{\frac{1}{\theta}} \frac{1}{1-\eta} \Gamma \left( 1 - \frac{1}{\theta} \cdot \frac{1}{1-\eta} \right). 
\] (34)

Substituting this expression into (29) and rearranging leads to the last result of the proposition.

**Proof of Proposition 3. Occupational Wage Gaps**

The proof of this proposition is straightforward given the results of Proposition 1.

**References**


### Table 16: Sample Statistics By Census Year

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<tbody>
<tr>
<td>Sample Size</td>
<td>641,686</td>
<td>694,419</td>
<td>4,057,685</td>
<td>4,711,405</td>
<td>5,216,431</td>
<td>3,147,547</td>
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<tr>
<td>Share of White Men in Sample</td>
<td>0.432</td>
<td>0.433</td>
<td>0.435</td>
<td>0.435</td>
<td>0.431</td>
<td>0.431</td>
</tr>
<tr>
<td>Share of White Women in Sample</td>
<td>0.475</td>
<td>0.469</td>
<td>0.459</td>
<td>0.447</td>
<td>0.437</td>
<td>0.431</td>
</tr>
<tr>
<td>Share of Black Men in Sample</td>
<td>0.042</td>
<td>0.044</td>
<td>0.047</td>
<td>0.054</td>
<td>0.061</td>
<td>0.065</td>
</tr>
<tr>
<td>Share of Black Women in Sample</td>
<td>0.052</td>
<td>0.055</td>
<td>0.059</td>
<td>0.065</td>
<td>0.071</td>
<td>0.074</td>
</tr>
<tr>
<td>Relative Wage Gap White Women</td>
<td>-0.578</td>
<td>-0.590</td>
<td>-0.476</td>
<td>-0.372</td>
<td>-0.299</td>
<td>-0.259</td>
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<tr>
<td>Relative Wage Gap Black Men</td>
<td>-0.379</td>
<td>-0.389</td>
<td>-0.215</td>
<td>-0.158</td>
<td>-0.142</td>
<td>-0.150</td>
</tr>
<tr>
<td>Relative Wage Gap Black Women</td>
<td>-0.875</td>
<td>-0.705</td>
<td>-0.479</td>
<td>-0.363</td>
<td>-0.317</td>
<td>-0.313</td>
</tr>
</tbody>
</table>

**Note:**

**B  Data Appendix**
Table 17: Occupation Categories for our Base Occupational Specification

<table>
<thead>
<tr>
<th>Category</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Sector</td>
<td>Police, Guards</td>
</tr>
<tr>
<td>Executives, Administrative, and Managerial</td>
<td>Food Preparation and Service</td>
</tr>
<tr>
<td>Management Related</td>
<td>Health Service</td>
</tr>
<tr>
<td>Architects</td>
<td>Cleaning and Building Service</td>
</tr>
<tr>
<td>Engineers</td>
<td>Personal Service</td>
</tr>
<tr>
<td>Math and Computer Science</td>
<td>Farm Managers</td>
</tr>
<tr>
<td>Natural Science</td>
<td>Farm Non-Managers</td>
</tr>
<tr>
<td>Health Diagnosing</td>
<td>Related Agriculture</td>
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<tr>
<td>Health Assessment</td>
<td>Forest, Logging, Fishers, and Hunters</td>
</tr>
<tr>
<td>Therapists</td>
<td>Vehicle Mechanic</td>
</tr>
<tr>
<td>Teachers, Postsecondary</td>
<td>Electronic Repairer</td>
</tr>
<tr>
<td>Teachers, Non-Postsecondary</td>
<td>Misc. Repairer</td>
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<tr>
<td>Librarians and Curators</td>
<td>Construction Trade</td>
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<tr>
<td>Social Scientists and Urban Planners</td>
<td>Extractive</td>
</tr>
<tr>
<td>Social, Recreation, and Religious Workers</td>
<td>Precision Production, Supervisor</td>
</tr>
<tr>
<td>Lawyers and Judges</td>
<td>Precision Metal</td>
</tr>
<tr>
<td>Arts and Athletes</td>
<td>Precision Wood</td>
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<td>Health Technicians</td>
<td>Precision Textile</td>
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<tr>
<td>Engineering Technicians</td>
<td>Precision Other</td>
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<tr>
<td>Science Technicians</td>
<td>Precision Food</td>
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<tr>
<td>Technicians, Other</td>
<td>Plant and System Operator</td>
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<td>Sales, All</td>
<td>Metal and Plastic Machine Operator</td>
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<td>Secretaries</td>
<td>Metal and Plastic Processing Operator</td>
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<tr>
<td>Information Clerks</td>
<td>Woodworking Machine Operator</td>
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<tr>
<td>Records Processing, Non-Financial</td>
<td>Textile Machine Operator</td>
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<td>Records Processing, Financial</td>
<td>Printing Machine Operator</td>
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<tr>
<td>Office Machine Operator</td>
<td>Machine Operator, Other</td>
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<tr>
<td>Computer and Communication Equipment Operator</td>
<td>Fabricators</td>
</tr>
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<td>Mail Distribution</td>
<td>Production Inspectors</td>
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<tr>
<td>Scheduling and Distributing Clerks</td>
<td>Motor Vehicle Operator</td>
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<tr>
<td>Adjusters and Investigators</td>
<td>Non Motor Vehicle Operator</td>
</tr>
<tr>
<td>Misc. Administrative Support</td>
<td>Freight, Stock, and Material Handlers</td>
</tr>
<tr>
<td>Private Household Occupations</td>
<td></td>
</tr>
<tr>
<td>Firefighting</td>
<td></td>
</tr>
</tbody>
</table>

Note:
Table 18: Examples of Occupations within Our Base Occupational Categories

**Management Related Occupations**

- Accountants and Auditors
- Underwriters
- Other Financial Officers
- Management Analysts
- Personnel, Training, and Labor Relations Specialists
- Purchasing Agents and Buyers
- Construction Inspectors
- Management Related Occupations, N.E.C.

**Health Diagnosing Occupations**

- Physicians
- Dentists
- Veterinarians
- Optometrists
- Podiatrists
- Health Diagnosing Practitioners, N.E.C.

**Personal Service Occupations**

- Supervisors, personal service occupations
- Barbers
- Hairdressers and Cosmetologists
- Attendants, amusement and recreation facilities
- Guides
- Ushers
- Public Transportation Attendants
- Baggage Porters
- Welfare Service Aides
- Family Child Care Providers
- Early Childhood Teacher Assistants
- Child Care Workers, N.E.C.

Note:
Table 19: Occupation Categories for our Broad Occupation Classification

<table>
<thead>
<tr>
<th>Home Sector</th>
<th>Sales, All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Executives, Administrative, and Managerial</td>
<td>Administrative Support, Clerks, and Record Keepers</td>
</tr>
<tr>
<td>Management Related</td>
<td>Fire, Police, and Guards</td>
</tr>
<tr>
<td>Architects, Engineers, Math, and Computer Science</td>
<td>Private Household and Food, Cleaning, and Personal Services</td>
</tr>
<tr>
<td>Natural and Social Scientists, Recreation, Religious, Arts, and Athletes</td>
<td>Farm, Related Agriculture, Logging, Forest, Fishing, Hunters and Extraction</td>
</tr>
<tr>
<td>Doctors and Lawyers</td>
<td>Mechanics and Construction</td>
</tr>
<tr>
<td>Nurses, Therapists, and Other Health Services</td>
<td>Precision Manufacturing</td>
</tr>
<tr>
<td>Teachers, Postsecondary</td>
<td>Manufacturing Operators</td>
</tr>
<tr>
<td>Teachers, Non-Postsecondary and Librarians</td>
<td>Fabricators, Inspectors, and Material Handlers</td>
</tr>
<tr>
<td>Health and Science Technicians</td>
<td>Vehicle Operators</td>
</tr>
</tbody>
</table>

Note: