The Cross-section of Firms over the Business Cycle: New Facts and a DSGE Exploration

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Abstract

Using a unique German firm-level data set, this paper is the first to study the cyclical properties of the cross-sections of firm-level real value added and Solow residual innovations, as well as capital and employment adjustment. We find two new business cycle facts: 1) The cross-sectional standard deviation of firm-level innovations in the Solow residual, value added and employment is robustly and significantly countercyclical. 2) The cross-sectional standard deviation of firm-level investment is procyclical. We show that a heterogeneous-firm RBC model with quantitatively realistic countercyclical innovations in the firm-level Solow residual and non-convex adjustment costs calibrated to the non-Gaussian features of the steady state investment rate distribution, produces investment dispersion that positively comoves with the cycle, with a correlation coefficient of 0.65, compared to 0.61 in the data. We argue more generally that the cross-sectional business cycle dynamics impose tight empirical restrictions on structural parameters and stochastic properties of driving forces in heterogeneous-firm models, and are therefore paramount in the calibration of these models.

JEL Codes: E20, E22, E30, E32.

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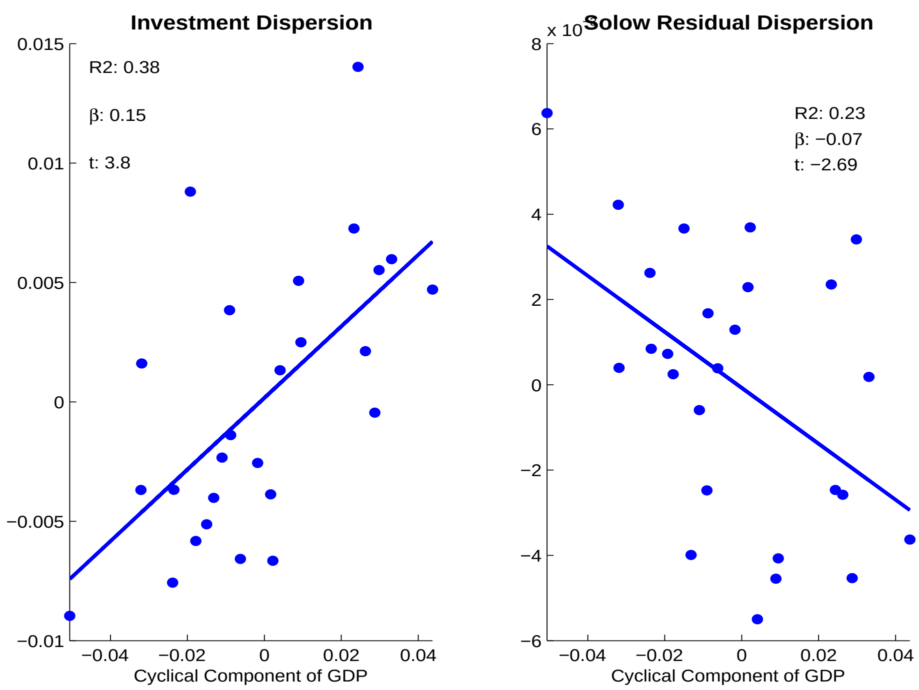
1 Introduction

The cross-section of firms – more specifically the dispersions of change rates of firm-level output, capital, employment and Solow residuals – display stark cyclical patterns. To the best of our knowledge, this is the first paper to systematically document the cyclical properties of these moments of the cross-section of firms. Using the balance sheet data set of Deutsche Bundesbank (USTAN) – a unique private sector, annual, firm-level data set that allows us to investigate 26 years of data (1973-1998), in which the cross-sections of the panel have over 30,000 firms per year on average –, we show that the cross-sectional standard deviations of the firm-level innovations in the Solow residual, value added and employment are robustly and significantly countercyclical. In contrast, the cross-sectional standard deviation of firm-level investment rates is robustly and significantly procyclical. These results are robust to different filtering methods for aggregate output, to using the cross-sectional interquartile range as a measure of dispersion, to using cyclical indicators other than aggregate output and to various changes in the sample selection criteria. Figure 1 illustrates these two new business cycle facts:

Figure 1: Cross-sectional Dispersion of Firm-Level Investment Rates and Solow Residual Innovations



It is clear that this finding is incompatible with a simple frictionless model of the firm with ex ante homogeneous firms and one idiosyncratic driving force, as this would imply that the stochastic properties of the driving force – dispersion in the innovations to firm-level Solow residuals – are at least qualitatively inherited by the outcome variables – dispersion of the firm-level investment rates. We propose a heterogenous-firm RBC model with persistent idiosyncratic productivity shocks and lumpy capital adjustment to explain both qualitatively and quantitatively the procyclicality of investment dispersion, even in the presence of countercyclical second-moment shocks in the driving force. The basic intuition, why lumpy capital adjustment is at least qualitatively a suitable candidate to explain this fact can be glanced from the simple Ss-model in Caplin and Spulber (1987):

Fact:

In a one-sided Ss-model a la Caplin and Spulber with a uniform gap-distribution, fixed optimal adjustment policy $S-s$ and shock Δz, the standard deviation of adjustments is increasing in Δz if and only if the fraction of adjusters is smaller than 0.5.

Proof:

As is well known, average adjustment in this environment is Δz. From this, it follows immediately that the standard deviation of adjustment is: $(0-\Delta z)^2\left(1-\frac{\Delta z}{S-s}\right)+\left((S-s)-\Delta z\right)^2\left(\frac{\Delta z}{S-s}\right)=\Delta z(S-s-\Delta z)$, which is increasing in Δz if and only if $\frac{\Delta z}{S-s}<0.5$, where $\frac{\Delta z}{S-s}$ is the fraction of adjusters.

This example shows that with sufficient inertia the extensive margin effect – and in this simple model all the dynamics are driven by the extensive margin effect, as the intensive margin of adjustment, $S-s$, is fixed by assumption – leads to a procyclical dispersion of adjustment. We will show that in a more realistic and realistically calibrated model this effect is still operative and can explain the observed procyclicality of investment dispersion almost exactly. We also provide further suggestive evidence that it is most likely lumpy capital adjustment that is generating this result: 1) we show that in sectors like manufacturing and construction, where we would expect non-convex factor adjustment to be most prevalent, procyclicality of investment dispersion is particularly pronounced; 2) we also show that in smaller firm sizes, i.e. firms that find themselves incapable of outgrowing adjustment costs, investment dispersion is significantly more procyclical than in the largest firm sizes. In contrast, conditional on firm size, finance variables do not seem to have a large impact on cyclicality of investment dispersion. We conclude from this that the explanation most likely does not lie in a financial friction. We also find no evidence of a composition effect in the sense that some large sectors or large firms have actually procyclical second-moment shocks that make the overall investment dispersion likewise procyclical.

Why is this important? First, in our view explaining the business cycle dynamics of the higher cross-sectional moments of the underlying macroeconomic aggregates is just as important for our understanding of the business cycle as explaining these aggregates themselves. A fully fledged business cycle theory has to speak to these cross-sectional dynamics as well. To the best of our knowledge, our paper is the first to systematically document these facts and explain the most striking of them: procyclical investment dispersion in the presence of countercyclical second-moment shocks. Secondly, heterogenous-firm models have seen increased use both in the macroeconomic as well as international finance literature. We show in this paper that cross-sectional dynamics impose tight restrictions on structural parameters as well as on the nature and stochastic properties of the driving forces in these models.[1] For instance, we show that procyclical investment dispersion in the presence of countercyclical second-moment shocks is only compatible with a strong capital-curvature of the revenue function of the firm, for there to be a strong enough procyclical extensive margin effect (see Gourio and Kashyap (2007) for a similar observation). We also document that the strengths of the countercyclical second-moment shocks must not be too strong to be compatible with procyclical investment dispersion. In particular, countercyclical second-moment shocks as large as suggested by Bloom (2008) and Bloom et al. (2008) and large enough to generate interesting business cycle dynamics are incompatible with this cross-sectional business cycle fact. That means cross-sectional dynamics have also strong implications for the nature of aggregate dynamics.

Related Literature

The empirical part of this paper is most closely related to a series of papers by Higson and Holly et al. (2002, 2004), Doepke and Holly et al. (2005, 2008), Doepke and Weber (2006), as well as Holly and Santoro (2008). Higson and Holly et al. (2002), using Compustat data, study empirically the cyclicality of the standard deviation, skewness and kurtosis of the sales growth rate distribution and find them to be countercyclical, countercyclical and procyclical, respectively. Higson and Holly et al. (2004) repeat this analysis for UK data on quoted firms, and Doepke and Holly et al. (2005) for Germany, using the USTAN database, with similar findings. Doepke and Weber (2006) study, again using USTAN data, the cyclicality of transitions between sales growth regimes in firm-level data. In contrast to these papers, we focus on the cyclicality of cross-sectional second moments only, but include value added, Solow residuals, investment rates and employment change rates into the analysis.[2] The quantitative part of this paper draws heavily on the recently developed literature on heterogenous-firm RBC models, developed in Kahn and Thomas (2008), Bachmann et al. (2008), Bloom (2008), Bloom et al. (2008) as well as Bachmann and Bayer (2009). Finally, our work is related to the work by Eisfeldt and Rampini (2005), who show that capital reallocation is procyclical and explain this in a two-sector model with costly capital reallocation.

[1] Kahn and Thomas (2005), in an earlier version of the 2008 paper, make a similar observation about the importance of general equilibrium in understanding cross-sectional firm dynamics. We confirm their conjecture here.

[2] Holly and Santoro (2008) as well as Doepke and Holly et al. (2008) start from the aforementioned empirical work and explore them in a monopolistically competitive model with financial frictions – the former – and in a monopolistically competitive model with simple Calvo-type price-stickiness – the latter.

2 The Facts

In Section 2.1 we briefly describe the USTAN data set and the main sample selection criteria we use. Details are relegated to Appendix A. In Section 2.2 we present the baseline facts: the contemporaneous correlations of cyclical aggregate output and the cross-sectional standard deviations of firm-level Solow residual and real value added innovations as well as employment change rates are negative, while the contemporaneous correlation of cyclical aggregate output and the cross-sectional standard deviation of firm-level investment rates is positive. In Section 2.3 we perform extensive robustness checks and also show, how these facts depend on observable firm characteristics.

2.1 A Brief Data Description

2.1.1 USTAN Data

USTAN is a large annual firm-level balance sheet data base (*Unternehmensbilanzstatistik*) collected by *Deutsche Bundesbank*. It is unique in its size and coverage. It provides annual firm level data from 1971 to 1998 from the balance sheets and the profit and loss accounts of over 60,000 firms per year (see Stoess (2001) and von Kalckreuth (2003) for further details). In the days when the discounting of commercial bills were one of the principal instruments of German monetary policy, Bundesbank law required the Bundesbank to assess the creditworthiness of all parties backing a commercial bill put up for discounting. The Bundesbank implemented this regulation by requiring balance sheet data of all parties involved. These balance sheet data were then archived and collected into a database.

Although the sampling design – one's commercial bill being put up for discounting – does not lead to a perfectly representative selection of firms in a statistical sense, the coverage of the sample is very broad. USTAN covers incorporated firms as well as privately-owned companies, which sets it apart from Compustat data.[3] Its sectoral coverage – while still somewhat biased to manufacturing firms – includes the construction, the service as well as the primary sectors. This makes it different from, for instance, the Annual Survey of Manufacturing (ASM) in the U.S.[4] The following table 1 displays the sectoral coverage of our final baseline sample.

Moreover, while there remains a bias to somewhat larger and financially healthier firms, the size coverage is still fairly broad: 31% of all firms in our final baseline sample have less than 20 employees and 57% have less than 50 employees (see Appendix A for details). Finally,

[3] Davis et al. (2006) show that studying only publicly traded firms can lead to wrong conclusions, in particular when higher cross-sectional moments are investigated.

[4] An additional advantage of these data is easy access: while access is on-site, it is practically free for researchers, so that results derived from this data base can be easily tested and checked.

Table 1: SECTORAL COVERAGE

1-digit Sector	Firm-year observations	Percentage
Agriculture	12,291	1.44
Mining & Energy	4,165	0.49
Manufacturing	405,787	47.5
Construction	54,569	6.39
Trade (Retail & Wholesale)	355,208	41.59
Transportation & Communication	22,085	2.59

the Bundesbank itself frequently uses the USTAN data for its macroeconomic analyzes and for cross-checking national accounting data. We take this as an indication that the bank considers the data sufficiently representative and of sufficiently high quality.[5] This makes the USTAN data a uniquely suitable data source for the study of cross-sectional business cycle dynamics.

2.1.2 Selection of the Baseline Sample

From the original USTAN data, we select only firms that report complete information on payroll, gross value added and capital stocks. Moreover, we drop observations from East German firms to avoid a break of the series in 1990.[6] In addition, we remove observations that stem from irregular accounting statements, e.g. when filing for bankruptcy or when closing operations. We deflate all but the capital data by the implicit deflator for gross value added from the German national accounts.

Capital is deflated with one-digit sector- and capital-good specific investment good price deflators within a perpetual inventory method. Even though USTAN data can be considered as particularly high quality data, we cannot directly use capital stocks as reported. Tax motivated depreciation and price developments of capital goods lead to a general understatement of the stock of capital a firm holds. Thus, capital stocks have to be recalculated using a perpetual inventory method. Similarly, we recover the amount of labor inputs from wage bills, as information on the number of employees (as opposed to payroll data) is only updated infrequently for some companies (see Appendix A for details). Finally, the firm-level Solow residual has to be calculated from data on gross value added and factor inputs.

We remove outliers according to the following procedure: we calculate log changes in real gross value added, the Solow residual, real capital and employment and drop all observations

[5]For example, data entry of the balance sheet data is double-checked by a second Bundesbank staff member.

[6]We identify a West German firm as a firm that has a West German address or has no address information but enters the sample before 1990.

where a change falls outside a 3 standard deviations interval around the year-specific mean.[7] We also drop those firms for which we do not have at least five observations in first differences. This leaves us with a sample of 854,105 firm-year observations, which corresponds to observations on 72,853 firms, i.e. the average observation length of a firm in the sample is 11.7 years. The average number of firms in the cross-section of any given year is 32,850. We perform numerous robustness checks with respect to each of these choices: we use sectoral deflators for value added, an aggregate investment good price deflator, change the cut-off rule to 2.5 and 5 standard deviations and leave all firms in the sample with two and twenty observations in first differences, respectively. None of these choices change our baseline results (see Appendix B for details).

2.1.3 Calculating the Solow Residual and Factor Adjustments

We compute the firm-level Solow residual based on the following Cobb-Douglas production function in accordance with our model:

$$y_t = z_t \epsilon_t k_t^{\theta} n_t^{\nu}, \tag{1}$$

where ϵ_t is the firm-specific component and z_t is aggregate productivity. We assume that labor input n_t is immediately productive, whereas capital k_t is pre-determined and inherited from last period. In our main specification, we estimate the output elasticities of the production factor, ν and θ, as median shares of factor expenditures over gross value added within each industry.[8]

For factor adjustment, we use the symmetric adjustment rate definition proposed in Davis et al. (1996). We thus define firm-level investment rates as $\frac{I_{i,t}}{0.5*(K_{i,t}+K_{i,t+1})}$ and firm-level employment adjustment rates as $\frac{\Delta E_{i,t}}{0.5*(E_{i,t-1}+E_{i,t})}$. We use log-differences in the Solow residual to capture Solow residual innovations, as the persistence of firm-level Solow residuals exhibits behavior close to a unit root. We remove firm fixed and sectoral-year effects from these first-difference variables to focus on idiosyncratic fluctuations that do not capture differences in sectoral responses to aggregate shocks or permanent ex-ante heterogeneity between firms.

[7]This outlier removal is done after removing firm and sectoral fixed effects. Centering the outlier removal around the year mean is important to avoid artificial and countercyclical skewness of the respective distributions.

[8]To check the robustness of our results, we try alternative specifications with predefined elasticities common across sectors. We also change the timing assumption to include a predetermined employment stock, as well as immediate adjustment in both factors. All results are very robust to the alternative ways to generate the firm-specific Solow residual (for a detailed discussion, see Bachmann and Bayer, 2009).

2.1.4 Macro data

When combining this micro data with aggregate data, we have to take a stance on what sectoral aggregate we view as the empirical counterpart to our model. We chose to include firms from the following six sectors in our analysis: agriculture, mining and energy, manufacturing, construction, trade (both retail and wholesale) as well as the transportation and communication sector. This aggregate can be roughly characterized as the non-financial private business sector in Germany. Whenever we use the term aggregate in the following, we mean this sector.

German national accounting data per one-digit sector allow us to compute real value added, investment, capital and employment data for this sectoral aggregate, and therefore also an aggregate Solow residual. Our USTAN sample captures on average 70% of sectoral value added, 44% of sectoral investment, 71% of its capital stock and 49% of sectoral employment.

In addition to representing a large part of the non-financial private business sector in Germany, USTAN also represents its cyclical behavior very well, as the following Table 2 shows:

Table 2: CYCLICALITY OF CROSS-SECTIONAL AVERAGES

Cross-sectional Moment	$\rho(\cdot, HP(100) - Y)$
$mean(\frac{I_{i,t}}{0.5*(K_{i,t}+K_{i,t+1})})$	0.792
$mean(\Delta \log \epsilon_{i,t})$	0.592
$mean(\Delta \log y_{i,t})$	0.663
$mean(\frac{\Delta E_{i,t}}{0.5*(E_{i,t-1}+E_{i,t})})$	0.602

2.2 Main Facts

The following Table 3 presents the main new stylized facts about the cross-sectional dynamics of firms:

Table 3: CYCLICALITY OF CROSS-SECTIONAL DISPERSION

Cross-sectional Moment	$\rho(\cdot, HP(100) - Y)$	5%	95%	Frac. w. opposite sign
$\sigma(\frac{I_{i,t}}{0.5*(K_{i,t}+K_{i,t+1})})$	0.613	0.338	0.784	0.001
$\sigma(\Delta \log \epsilon_{i,t})$	-0.481	-0.678	-0.306	0.000
$\sigma(\Delta \log y_{i,t})$	-0.450	-0.675	-0.196	0.005
$\sigma(\frac{\Delta E_{i,t}}{0.5*(E_{i,t-1}+E_{i,t})})$	-0.498	-0.717	-0.259	0.001

The first column of Table 3 shows the contemporaneous correlation of the cyclical component of aggregate output[9] with the cross-sectional standard deviations of the firm-level in-

[9]For the baseline scenario we use log-output with an HP-parameter 100.

vestment rates, the percentage changes in the firm-level Solow residual and real value added as well as employment changes. The first is clearly procyclical, the latter three countercyclical. The next two columns show the 5% and 95% confidence bands from 10,000 parametric bootstrap simulations.[10] The last column displays the fraction of negative correlations for the standard deviation of the firm-level investment rates, and the fraction of positive correlations for the remaining three standard deviations in these bootstrap simulations. These three columns together show that the sign of all correlations is significant. This means that firm-level investment rates display *procyclical* dispersion, whereas the cross-sectional standard deviations of the Solow residual, output and employment changes are *countercyclical*. In the next section, we show that this finding is robust to the specific choices we have made in generating the numbers in Table 3.

2.3 Robustness

The following Table 4 shows that procyclical investment dispersion is robust to the choice of the cyclical indicator.[11] Specifically, we chose a smaller smoothing parameter for the HP filter, following Ravn and Uhlig (2001): 6.25. Furthermore, we apply a log-difference filter to output, use the linearly detrended average cross-sectional investment rate, and the HP(100)-filtered aggregate employment and Solow residuals as cyclical indicators. The result stands:

Table 4: PROCYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - ROBUSTNESS TO CYCLICAL INDICATOR

Cyclical Indicator	$\rho(\sigma(\frac{I_{i,t}}{0.5*(K_{i,t}+K_{i,t+1})}),\cdot)$
HP(6.25)-Y	0.529
Log-diff-Y	0.419
Average $\frac{I}{K}$	0.834
HP(100)-E	0.533
HP(100)-Solow Residual	0.511

Conversely, the following Table 5 shows that our finding is also robust to the numerous choices we have made for the other part of the correlation: one can use the interquartile range (IQR) as the dispersion measure, and one can study the firm level net percentage change in capital as opposed to the investment rate. Moreover, removing firm-level and sectoral fixed

[10] We use a pairwise unrestricted VAR with one lag as the parametric model. The results from a nonparametric overlapping block bootstrap with a block size of four are similar to the parametric bootstrap.

[11] This is also true for the three other variables, and for $\sigma(\Delta \log \epsilon_{i,t})$ and $\sigma(\frac{\Delta E_{i,t}}{0.5*(E_{i,t-1}+E_{i,t})})$, we have documented this and other robustness tests elsewhere: Bachmann and Bayer (2009).

effects does not induce this procyclicality, as row three of this table shows. Finally, the last two rows demonstrate that the result is neither driven by the German reunification, nor by the strong recession in 1975.

Table 5: PROCYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - MORE ROBUSTNESS

Cross-sectional Moment	$\rho(\cdot, HP(100)-Y)$
$IQR(\frac{I_{i,t}}{0.5*(K_{i,t}+K_{i,t+1})})$	0.647
$\sigma(\Delta \log k_{i,t})$	0.442
$\sigma(\frac{I_{i,t}}{0.5*(K_{i,t}+K_{i,t+1})})_{\text{raw}}$	0.653
$\sigma(\frac{I_{i,t}}{0.5*(K_{i,t}+K_{i,t+1})})_{1973-1990}$	0.538
$\sigma(\frac{I_{i,t}}{0.5*(K_{i,t}+K_{i,t+1})})_{1977-1998}$	0.539

We finish this section by showing how the cyclicality of cross-sectional investment dispersion is distributed both across the one-digit sectors we have data on as well as firm sizes. We use again the cross-sectional standard deviation of the firm-level investment rate and the HP(100)-filtered log-output of the sectoral aggregate as inputs into the correlation measure.

Table 6: CYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - SECTORS

1-digit Sector	$\rho(\sigma(\frac{I_{i,t}}{0.5*(K_{i,t}+K_{i,t+1})}), HP(100)-Y)$	$\rho(\sigma(\Delta \log \epsilon_{i,t}), HP(100)-Y)$
Agriculture	0.074	-0.045
Mining & Energy	0.063	-0.166
Manufacturing	0.509	-0.607
Construction	0.480	-0.483
Trade (Retail & Wholesale)	0.449	-0.192
Transportation & Communication	0.219	-0.036

Table 6 shows that procyclicality of investment dispersion is strongly prevalent in the goods-producing sectors, manufacturing and construction, as well as trade. The transportation and communication sector exhibits a much smaller effect, whereas in the primary sectors investment dispersion is nearly acyclical. To put these findings in perspective, we also display the cyclicality of the cross-sectional innovations-to-Solow-residual dispersion, which – despite the procyclicality of investment dispersion – is strongly countercyclical in the goods-producing sectors. Conversely, in the primary sectors both dispersions of driving forces and outcome variables are acyclical.

As Table 7 shows, procyclicality of investment dispersion is driven mainly by the smaller firms, independently of the size measure. Large firms, in contrast, display only weakly procycli-

Table 7: CYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - FIRM SIZE

Size Class / Criterion	Capital	Employment	Value Added
Smallest 33%	0.516	0.582	0.574
Smallest 75%	0.653	0.644	0.644
Smallest 95%	0.634	0.638	0.645
Largest 5%	0.182	0.109	0.012

cal to acyclical investment dispersion. This distinction is significant in the sense that at least if size is measured in terms of employment or value added, neither the point estimate for the smallest size class lies in the $[5\%, 95\%]-$bands of the largest size class nor vice versa. For capital, the point estimate for the smallest size class falls into the $[5\%, 95\%]-$bands of the largest size class, but not vice versa.[12]

Finally, the last Table 8 shows that conditional on firm size – as measured by capital – the financial situation of a firm – as measured by the equity-asset-ratio – hardly matters for the cyclicality of investment dispersion:

Table 8: CYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - FINANCIAL SITUATION

Equity-Asset-Ratio Tercile	Smallest 33% - Capital	Largest 5% - Capital
First	0.487	0.072
Second	0.292	0.068
Third	0.377	-0.151

Tables 6 to 8 together with the finding that the Solow residual processes for small and large firms hardly differ both on average over time and in terms of cyclicality of their innovations,[13] at least suggests that the friction necessary to explain the differential cyclicality of the dispersions of firm-level innovations-to-Solow-residual and investment rates, respectively, can neither be found in financial constraints nor in different shock processes. It does also not appear to be driven by a selection effect from large sectors. Instead, we will show that the presence of lumpy capital adjustment is a plausible cause for this aspect of the cross-sectional firm dynamics. Indeed, the fact that procyclical investment dispersion is mostly prevalent in the goods-producing sectors as well as in smaller firms, i.e. firms where we would a priori expect non-convexities in the adjustment technology to be more relevant, is at least consistent with our explanation.

[12]See Appendix A for detailed information on the size distribution of firms in our sample.
[13]See Bachmann and Bayer (2009) for an in-depth discussion of this issue.

3 The Model

In this section we describe our model economy. We start with the firm's problem, followed by a brief description of the households and the definition of equilibrium. We conclude with a sketch of the equilibrium computation. We follow closely Kahn and Thomas (2008) and Bachmann et al. (2008). Since there the model set up is discussed in detail, we will be rather brief here.

The main departure from either models is the introduction of a second exogenous aggregate state, the standard deviation of the current idiosyncratic shock distribution: $\sigma(\epsilon_t)$. The motivation for this is both realism, as we find these second-moment shocks in the data, but also conservatism: we will show in Section 5.1 that without countercyclical second-moment shocks even with very small fixed costs to adjustment the investment rate dispersion is very procyclical, even more procyclical than in the data. This comes as no surprise, as without countercyclical second-moment shocks there is no countervailing force that would undo the extensive margin effect that in turn causes the investment rate dispersion to be procyclical. Thus, since this is a quantitative exercise using the correct amount of second-moment countercyclicality is important. Following Kahn and Thomas (2008), we approximate this now bivariate aggregate state process with a discrete Markov chain.

3.1 Firms

The economy consists of a unit mass of small firms. We do not model entry and exit decisions. There is one commodity in the economy that can be consumed or invested. Each firm produces this commodity, employing its pre-determined capital stock (k) and labor (n), according to the following Cobb-Douglas decreasing-returns-to-scale production function ($\theta > 0$, $\nu > 0$, $\theta + \nu < 1$):

$$y_t = z_t \epsilon_t k_t^{\theta} n_t^{\nu}, \tag{2}$$

where z_t and ϵ_t denote aggregate and firm-specific (idiosyncratic) technology, respectively.

The idiosyncratic technology process has autocorrelation ρ_I. It follows a Markov chain, whose transition matrix depends on the aggregate state of its time-varying standard deviation, $\sigma(\epsilon_t)$. In contrast, its support is independent of the aggregate state. To also capture observed excess kurtosis in the idiosyncratic productivity shocks, we use a mixture of two Gaussian distributions in the Tauchen-approximation algorithm instead of the usual normal distribution.[14]

We denote the trend growth rate of aggregate productivity by $(1-\theta)(\gamma-1)$, so that aggregate y and k grow at rate $\gamma - 1$ along the balanced growth path. From now on we work with k and

[14] Tauchen (1986). For details, see Section 4.

y (and later C) in efficiency units. The linearly detrended logarithm of aggregate productivity levels, which we also denote by z, as well as linearly detrended $\sigma(\epsilon)$ evolve according to a VAR(1) process, with normal innovations v that have zero mean and covariance Ω:

$$\begin{pmatrix} \log z_t \\ \sigma(\epsilon_t) - \bar{\sigma}(\epsilon) \end{pmatrix} = \rho_A \begin{pmatrix} \log z_{t-1} \\ \sigma(\epsilon_{t-1}) - \bar{\sigma}(\epsilon) \end{pmatrix} + v_t, \tag{3}$$

where $\bar{\sigma}(\epsilon)$ denotes the steady state standard deviation of idiosyncratic productivity innovations.[15]

Productivity innovations at different aggregation levels are independent. Also, idiosyncratic productivity shocks are independent across productive units. In contrast, we do not impose any restrictions on Ω.

Each period a firms draws from a time-invariant distribution, G, its current cost of capital adjustment, $\xi \geq 0$, which is denominated in units of labor. G is a uniform distribution on $[0, \bar{\xi}]$, common to all firms. Draws are independent across firms and over time, and employment is freely adjustable.

At the beginning of a period, a firm is characterized by its pre-determined capital stock, its idiosyncratic productivity, and its capital adjustment cost. Given the aggregate state, it decides its employment level, n, production and depreciation occurs, workers are paid, and investment decisions are made. Then the period ends.

Upon investment, i, the firm incurs a fixed cost of $\omega\xi$, where ω is the current real wage rate. Capital depreciates at a rate δ. We can then summarize the evolution of the firm's capital stock (in efficiency units) between two consecutive periods, from k to k', as follows:

	Fixed cost paid	$\gamma k'$
$i \neq 0$:	$\omega\xi$	$(1-\delta)k + i$
$i = 0$:	0	$(1-\delta)k$

Given the i.i.d. nature of the adjustment costs, it is sufficient to describe differences across firms and their evolution by the distribution of firms over (ϵ, k). We denote this distribution by μ. Thus, $\left(z, \sigma(\epsilon), \mu\right)$ constitutes the current aggregate state and μ evolves according to the law of motion $\mu' = \Gamma(z, \sigma(\epsilon), \mu)$, which firms take as given.

Next we describe the dynamic programming problem of each firm. We will take two shortcuts (details can be found in Kahn and Thomas, 2008). First, we state the problem in terms of

[15] Specifying this process in terms of $\log(\sigma(\epsilon))$, in order to avoid negativity of the standard deviation of idiosyncratic productivity shocks is – given its relatively low variability (see Bachmann and Bayer, 2009) – an unnecessary precaution that does not change the results.

utils of the representative household (rather than physical units), and denote by $p = p(z, \sigma(\epsilon), \mu)$ the marginal utility of consumption. This is the relative intertemporal price faced by a firm. Second, given the i.i.d. nature of the adjustment costs, continuation values can be expressed without explicitly taking into account future adjustment costs.

Let $V^1(\epsilon, k, \xi; z, \sigma(\epsilon), \mu)$ denote the expected discounted value—in utils—of a firm that is in idiosyncratic state (ϵ, k, ξ), given the aggregate state $(z, \sigma(\epsilon), \mu)$. Then the expected value prior to the realization of the adjustment cost draw is given by:

$$V^0(\epsilon, k; z, \sigma(\epsilon), \mu) = \int_0^{\bar{\xi}} V^1(\epsilon, k, \xi; z, \sigma(\epsilon), \mu) G(d\xi). \tag{4}$$

With this notation the dynamic programming problem is given by:

$$V^1(\epsilon, k, \xi; z, \sigma(\epsilon), \mu) = \max_n \{\text{CF} + \max(V_i, \max_{k'} [-\text{AC} + V_a])\}, \tag{5}$$

where CF denotes the firm's flow value, V_i the firm's continuation value if it chooses inaction and does not adjust, and V_a the continuation value, net of adjustment costs AC, if the firm adjusts its capital stock. That is:

$$\text{CF} = [z\epsilon k^\theta n^\nu - \omega(z, \sigma(\epsilon), \mu) n] p(z, \sigma(\epsilon), \mu), \tag{6a}$$

$$V_i = \beta \text{E}[V^0(\epsilon', (1-\delta)k/\gamma; z', \sigma(\epsilon)', \mu')], \tag{6b}$$

$$AC = \xi \omega(z, \sigma(\epsilon), \mu) p(z, \sigma(\epsilon), \mu), \tag{6c}$$

$$V_a = -i\, p(z, \sigma(\epsilon), \mu) + \beta \text{E}[V^0(\epsilon', k'; z', \sigma(\epsilon)', \mu')], \tag{6d}$$

where both expectation operators average over next period's realizations of the aggregate and idiosyncratic shocks, conditional on this period's values, and we recall that $i = \gamma k' - (1-\delta)k$. Also, β denotes the discount factor of the representative household.

Taking as given intra- and intertemporal prices $\omega(z, \sigma(\epsilon), \mu)$ and $p(z, \sigma(\epsilon), \mu)$, and the law of motion $\mu' = \Gamma(z, \sigma(\epsilon), \mu)$, the firm chooses optimally labor demand, whether to adjust its capital stock at the end of the period, and the optimal capital stock, conditional on adjustment. This leads to policy functions: $N = N(\epsilon, k; z, \sigma(\epsilon), \mu)$ and $K = K(\epsilon, k, \xi; z, \sigma(\epsilon), \mu)$. Since capital is predetermined, the optimal employment decision is independent of the current adjustment cost draw.

3.2 Households

We assume a continuum of identical households that have access to a complete set of state-contingent claims. Hence, there is no heterogeneity across households. Moreover, they own shares in the firms and are paid dividends. We do not need to model the household side explicitly (see Kahn and Thomas (2008) for details), and concentrate instead on the first-order conditions to determine the equilibrium wage and the intertemporal price.

Households have a standard felicity function in consumption and (indivisible) labor:

$$U(C, N^h) = \log C - AN^h, \tag{7}$$

where C denotes consumption and N^h the household's labor supply. Households maximize the expected present discounted value of the above felicity function. By definition we have:

$$p(z, \sigma(\epsilon), \mu) \equiv U_C(C, N^h) = \frac{1}{C(z, \sigma(\epsilon), \mu)}, \tag{8}$$

and from the intratemporal first-order condition:

$$\omega(z, \sigma(\epsilon), \mu) = -\frac{U_N(C, N^h)}{p(z, \sigma(\epsilon), \mu)} = \frac{A}{p(z, \sigma(\epsilon), \mu)}. \tag{9}$$

3.3 Recursive Equilibrium

A *recursive competitive equilibrium* is a set of functions

$$\left(\omega, p, V^1, N, K, C, N^h, \Gamma\right),$$

that satisfy

1. *Firm optimality*: Taking ω, p and Γ as given, $V^1(\epsilon, k; z, \sigma(\epsilon), \mu)$ solves (5) and the corresponding policy functions are $N(\epsilon, k; z, \sigma(\epsilon), \mu)$ and $K(\epsilon, k, \xi; z, \sigma(\epsilon), \mu)$.

2. *Household optimality*: Taking ω and p as given, the household's consumption and labor supply satisfy (8) and (9).

3. *Commodity market clearing*:

$$C(z, \sigma(\epsilon), \mu) = \int z\epsilon k^{\theta} N(\epsilon, k; z, \sigma(\epsilon), \mu)^{\nu} d\mu - \int\int_0^{\bar{\xi}} [\gamma K(\epsilon, k, \xi; z, \sigma(\epsilon), \mu) - (1-\delta)k] dG d\mu.$$

4. *Labor market clearing*:

$$N^h(z,\sigma(\epsilon),\mu) = \int N(\epsilon,k;z,\sigma(\epsilon),\mu)d\mu + \int\int_0^{\bar{\xi}} \xi \mathscr{J}\Big(\gamma K(\epsilon,k,\xi;z,\sigma(\epsilon),\mu) - (1-\delta)k\Big)dGd\mu,$$

   where $\mathscr{J}(x) = 0$, if $x = 0$ and 1, otherwise.

5. *Model consistent dynamics*: The evolution of the cross-section that characterizes the economy, $\mu' = \Gamma(z,\sigma(\epsilon),\mu)$, is induced by $K(\epsilon,k,\xi;z,\sigma(\epsilon),\mu)$ and the exogenous processes for z, $\sigma(\epsilon)$ as well as ϵ.

Conditions 1, 2, 3 and 4 define an equilibrium given Γ, while step 5 specifies the equilibrium condition for Γ.

3.4 Solution

As is well-known, (5) is not computable, since μ is infinite dimensional. Hence, we follow Krusell and Smith (1997, 1998) and approximate the distribution μ by its first moment over capital, and its evolution, Γ, by a simple log-linear rule. In the same vein, we approximate the equilibrium pricing function by a log-linear rule discrete – aggregate state by discrete aggregate state:

$$\log \bar{k}' = a_k\big(z,\sigma(\epsilon)\big) + b_k\big(z,\sigma(\epsilon)\big)\log \bar{k}, \tag{10a}$$

$$\log p = a_p\big(z,\sigma(\epsilon)\big) + b_p\big(z,\sigma(\epsilon)\big)\log \bar{k}, \tag{10b}$$

where $\bar{k}$ denotes aggregate capital holdings. Given (9), we do not have to specify an equilibrium rule for the real wage. As usual with this procedure, we posit this form and check that in equilibrium it yields a good fit to the actual law of motion. In contrast to models without second moment shocks, where it has been extensively shown that the first moment suffices, we show here that the pure R^2 goodness-of-fit metric does not perform as well anymore: R^2 below 0.9 are possible, as we shall see in Section 5.2. Nevertheless, Bachmann and Bayer (2009) show that the aggregate dynamics of such an economy are hardly affected, when higher moments of the capital distribution are included and the R^2 are pushed closer to unity (see Bachmann et al. (2008) for a similar observation). We show here that also the cross-sectional dynamics are affected only to a small degree. And since we have consistently found that not including higher moments will lead to a slight underestimation of the procyclicality of investment dispersion, we prefer the increase in computational speed and report our results, unless otherwise noted, with the first moment only as a state variable.

Combining these assumptions and substituting $\bar{k}$ for μ into (5) and using (10a)–(10b), we have that (5) becomes a computable dynamic programming problem with policy functions $N = N(\epsilon, k; z, \sigma(\epsilon), \bar{k})$ and $K = K(\epsilon, k, \xi; z, \sigma(\epsilon), \bar{k})$. We solve this problem via value function iteration on V^0.

With these policy functions, we can then simulate a model economy *without* imposing the equilibrium pricing rule (10a), but rather solve for it along the way. We simulate the model economy for 1,600 time periods and discard the first 100 observations, when computing any statistics. This procedure generates a time series of $\{p_t\}$ and $\{\bar{k}_t\}$ endogenously, with which assumed rules (10a)–(10b) can be updated via a simple OLS regression. The procedure stops when the updated coefficients $a_k\big(z, \sigma(\epsilon)\big)$ and $b_k\big(z, \sigma(\epsilon)\big)$, as well as $a_p\big(z, \sigma(\epsilon)\big)$ and $b_p\big(z, \sigma(\epsilon)\big)$ are sufficiently close to the previous ones. We skip the details of this procedure, as this has been outlined elsewhere – see Kahn and Thomas (2008) and Bachmann et al. (2008).

4 Calibration

The model period for the baseline model is a year – in congruence with our data frequency. The following parameters have standard values: $\beta = 0.98$ and $\delta = 0.094$, which we compute from German national accounting data for the sectoral aggregate that the USTAN sample corresponds to: the non-financial private business sector. Given this depreciation rate, we pick $\gamma = 1.014$, in order to match the time-average aggregate investment rate of 0.108. This number is also consistent with German long-run growth rates. The log-felicity function features an elasticity of intertemporal substitution (EIS) of one. The disutility of work parameter, A, is chosen to generate an average time spent at work of 0.33: $A = 2$ for the baseline calibration.

We set the output elasticities of labor and capital to $\nu = 0.5565$ and $\theta = 0.2075$, respectively, which correspond to the measured average labor and capital shares in manufacturing in the USTAN data base. While our data also include a considerable amount of firms from other sectors, any weighted average or median of these shares would still be close to the manufacturing values, which is why we decided to use them in our baseline calibration. We discuss robustness to this parameter choice in Section 5.1.[16]

Next, we discuss the parameters of the two-state aggregate shock process. Here we simply

[16] If one views the DRTS assumption as a mere stand-in for a CRTS production function with monopolistic competition, than these choices would correspond to an employment elasticity of the underlying production function of 0.7284 and a markup of 31%. Given the regulated product markets in Germany, this is a reasonable value. The implied capital elasticity of the revenue function, $\frac{\theta}{1-\nu}$ is 0.47. Finally, model simulations show that using the capital share as an estimate for the output elasticity of capital under the null hypothesis of the model leads to a small overestimation of the latter, which, as we will show in Section 5.1, leads to our the baseline calibration being conservative relative to our main result: procyclicality of investment dispersion.

estimate a bivariate, unrestricted VAR with the linearly detrended natural logarithm of the aggregate Solow residual[17] and the linearly detrended $\sigma(\epsilon)$-process from the USTAN data.[18] The parameters of this VAR are as follows:[19]

$$\rho_A = \begin{pmatrix} 0.3144 & 0.051 \\ -2.3775 & 0.7794 \end{pmatrix} \quad \Omega = \begin{pmatrix} 0.0176 & -0.5773 \\ -0.5773 & 0.0027 \end{pmatrix} \tag{11}$$

This process is discretized on a $[5 \times 5]-$grid, using a bivariate analog of Tauchen's procedure.

We measure the steady state standard deviation of idiosyncratic technology innovations, corresponding to the baseline choices for ν and θ, as $\bar{\sigma}(\epsilon) = 0.1201$. Since these innovations also exhibit mild excess kurtosis – 4.4480 on average over our time horizon –,[20] and since the adjustment cost parameter $\bar{\xi}$ will be identified by the kurtosis of the firm-level investment rate (next to its skewness), we want to avoid attributing excess kurtosis in the firm-level investment rate to nonlinearities in the adjustment technology, when the driving force itself has kurtosis. Hence, we incorporate the measured excess kurtosis into the discretization process for the idiosyncratic technology state.[21] Finally, we set $\rho_I = 0.95$, in accordance with the high persistence of Solow residual innovations we find in the data. This process is discretized on a $19-$state-grid, using the Tauchen's procedure with mixed Gaussian normals.[22]

Given the aforementioned set of parameters $\left(\beta, \delta, \gamma, A, \nu, \theta, \rho_A, \Omega, \bar{\sigma}(\epsilon), \rho_I\right)$[23], we then calibrate the adjustment costs parameter $\bar{\xi}$ to minimize a quadratic form in the logarithmic differences between the time-average firm-level investment rate skewness produced by the model and the data, as well as the time-average firm-level investment rate kurtosis:

$$\min_{\bar{\xi}} \Psi(\bar{\xi}) \equiv 0.5 \cdot \Bigg[\Bigg(\log\Big(\frac{1}{26}\sum_t skewness(\frac{I_{i,t}}{0.5 * (K_{i,t} + K_{i,t+1})})(\bar{\xi}) - 1.6645\Big)\Bigg)^2 + \Bigg(\log\Big(\frac{1}{26}\sum_t kurtosis(\frac{I_{i,t}}{0.5 * (K_{i,t} + K_{i,t+1})})(\bar{\xi}) - 19.1046\Big)\Bigg)^2 \Bigg]. \tag{12}$$

As can be seen from (12), the histogram of firm-level investment rates exhibits both sub-

[17]We use again $\nu = 0.5565$ and $\theta = 0.2075$ in these calculations.

[18]After firm-level and sectoral fixed effects have been removed, as described in Section 2.1.

[19]With a slight abuse of notation, but for the sake of readability, Ω displays standard deviations on the main diagonal and correlations on the off diagonal.

[20]We find no skewness.

[21]We achieve this by using a mixture of two Gaussian distributions: $N(0, 0.0777)$ and $N(0, 0.1625)$ – the standard deviations are 0.1201 ± 0.0424 – with a weight of 0.4118 on the first distribution. As will become clear shortly, using a Gaussian process would only lead to an increase in calibrated adjustment costs, and hence a strengthening of our main result.

[22]The cross-sectional results do not change significantly with either an increase in the fineness of the aggregate grid to $[9 \times 9]$, nor with one in the idiosyncratic grid to a $35-$state-grid.

[23]Plus the mixing parameter.

stantial positive skewness – 1.6645 – as well as excess kurtosis – 19.1046. Caballero et al. (1995) document a similar fact for U.S. manufacturing plants. They also argue that non-convex capital adjustment costs are an important ingredient to explain such a strongly non-Gaussian distribution, given a close-to-Gaussian shock process. We therefore, use the deviation from Gaussianity in firm-level investment rates to identify $\bar{\xi}$.

The following Table 9 demonstrates identification of $\bar{\xi}$, as cross-sectional skewness and kurtosis of the firm-level investment rates are both monotonically increasing in $\bar{\xi}$. We pick $\bar{\xi} = 0.25$ as our baseline case.[24] A description of the aggregate dynamics of the baseline calibration – which are not the focus of this paper – is relegated to Appendix C.

Table 9: CALIBRATION OF ADJUSTMENT COSTS - $\bar{\xi}$

$\bar{\xi}$	Skewness	Kurtosis	$\Psi(\bar{\xi})$	Adj. costs/ Unit of Output
0.01	0.7851	5.0429	1.0814	1.5%
0.05	1.5171	7.6509	0.6504	4.2%
0.10	1.9350	9.3411	0.5170	6.8%
0.25 (BL)	2.5623	12.1704	0.4413	13.3%
0.5	3.0723	14.7831	0.4698	23.3%
1	3.5970	17.8299	0.5471	43.2%

5 Results

5.1 Baseline Results

Figure 2 shows that indeed the model produces procyclical investment dispersion close to the one found in the data and shown in Figure 1 in the introduction. We choose the HP(100)-filtered aggregate model output as our cyclical measure. Table 10 summarizes our main result numerically: in our baseline calibration the model matches the procyclicality of firm-level investment rate dispersion almost exactly, and the countercyclical dispersions of sales and employment changes are captured at least to a large extent.[25]

The next Table 11 illustrates how lumpy capital adjustment and countercyclical second moment shocks interact to generate the procyclicality result.

[24] We searched over a much finer grid of $\bar{\xi}$ than displayed in the table, in order to find the optimal $\bar{\xi}$.

[25] These numbers are obtained from a longer simulation of $T = 1500$. Using an even longer simulation and breaking it up into pieces of 26 independent time series produces an average value of 0.700 for the correlation between investment rate dispersion and cyclical output with a standard deviation of: 0.106. The range is $[0.423, 0.861]$.

Figure 2: Cross-sectional Dispersion of Firm-Level Investment Rates and Solow Residual Innovations

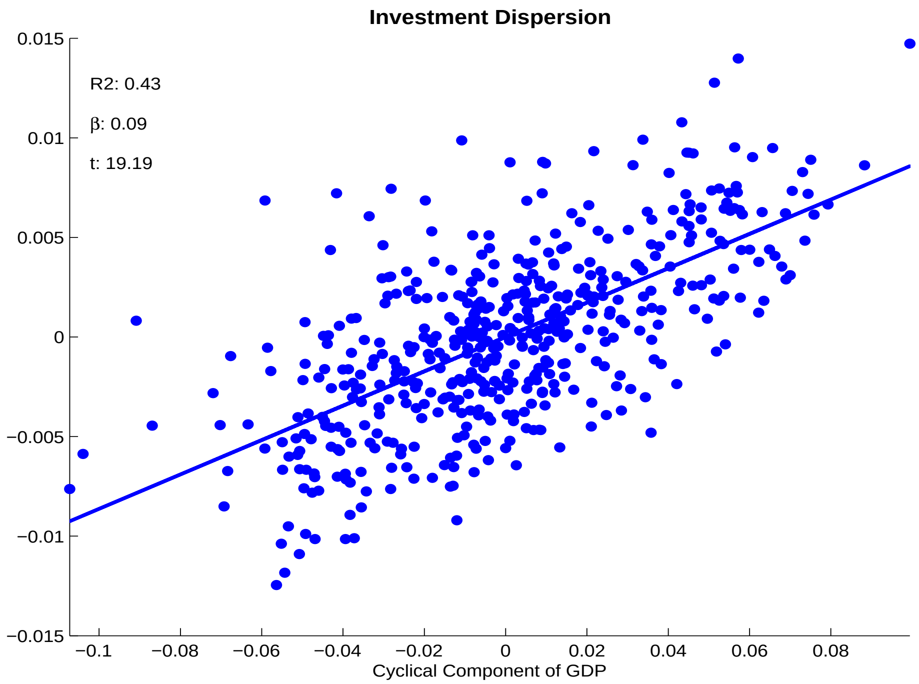


Table 10: CYCLICALITY OF CROSS-SECTIONAL DISPERSION - BASELINE MODEL

Cross-sectional Moment	Data	Model
$\sigma(\frac{I_{i,t}}{0.5*(K_{i,t}+K_{i,t+1})})$	0.613	0.652
$\sigma(\Delta \log y_{i,t})$	-0.450	-0.287
$\sigma(\frac{\Delta E_{i,t+1}}{0.5*(E_{i,t}+E_{i,t+1})})$	-0.498	-0.292

Table 11: ADJUSTMENT COSTS AND CYCLICALITY OF INVESTMENT DISPERSION

$\bar{\xi}$	Full Model w. 2nd moment shocks	Model w/o. 2nd moment shocks
0	-0.3845	-0.6319
0.06 (skewness only)	0.2232	0.8556
0.10	0.3795	0.8611
0.25 (BL)	0.6517	0.8740
0.5	0.7913	0.8834
1	0.8738	0.8958

Two findings are important: in the presence of countercyclical second moment shocks, the procyclicality of investment dispersion is a smooth monotonically increasing function of the adjustment cost parameter. What is perhaps surprising is that the level of adjustment costs that best matches the cross-sectional average skewness and kurtosis of firm-level investment rates – two statistics that have been known to be related to the level of nonconvexities at the micro-level (see Caballero et al., 1995) – also leads to the model matching almost exactly an important time series moment of the cross-sectional business cycle dynamics. The table also shows that a more conservative calibration that calibrates to the cross-sectional skewness of firm-level investment rates only and puts zero weight on their kurtosis, still generates a sizeable level of procyclicality in investment dispersion, given that the frictionless case, unsurprisingly, merely replicates the countercyclicality of the dispersion of the driving force.

Moreover, the second column of this table shows that without second moment shocks, a minimal level of non-convexity immediately generates procyclicality in investment dispersion, as shown in the introduction. But it also makes the model overshoot this number considerably. Thus, countercyclical second moment shocks are an important part in understanding cross-sectional firm dynamics, both in generating countercyclical dispersions of sales and employment changes, but also to generate realistic procyclicality in investment dispersion. Without them, it would simply be too easy to generate the latter. We view this as an important confirmation of our calibration and our mechanism: in the presence of quantitatively realistic countercyclicality of the dispersion of the driving force, it is exactly that level of adjustment costs that matches best the nonlinear average moments of the investment rate distribution that also generates just the right correlation coefficient between the standard deviation of investment rates and aggregate output. Table 11 shows that this identification is rather tight.

Table 12 illustrates the simple extensive margin mechanism sketched in the introduction and how lumpy capital adjustment and the curvature of the revenue function in capital – through procyclicality of the extensive margin, i.e. the fraction of firms that adjust – interact to generate the procyclicality result:

Table 12: FACTOR ELASTICITIES AND CYCLICALITY OF INVESTMENT DISPERSION

Cross-sectional Moment	Baseline (0.47)	Rev. Ela.=0.57	Rev. Ela.=0.63
$\sigma(\frac{I_{i,t}}{0.5*(K_{i,t}+K_{i,t+1})})$	0.6517	0.1745	-0.3267
Fraction of Adjusters	0.6413	-0.0549	-0.4686

To generate the results in columns two and three we set (ν = 0.5333, θ = 0.2667) and (ν = 0.5556, θ = 0.2778), respectively, compared to (ν = 0.5565, θ = 0.2075) in the baseline scenario.[26] It is clear that larger revenue elasticities in capital after labor has been maximized out, imply a lower procyclicality of the extensive margin and thus for the investment rate dispersion. Smaller revenue elasticities or higher curvature of the production function imply that the intensive margin of investment becomes less flexible: the range of the optimal capital return level in the baseline scenario is $[0.0275, 34.8021]$, for the second column $[0.0188, 85.3812]$ and $[0.0069, 148.9601]$ for the third column; all three for the same process for idiosyncratic technology. To achieve the optimal path for aggregate investment, the extensive margin becomes more important for the firms, the higher the curvature of the revenue function. This effect of curvature is well known and has been explained in detail in Gourio and Kashyap (2007). Table 13 shows the effect of general equilibrium on both the procyclicality of the extensive margin as well as the procyclicality of investment dispersion. Real wage and interest rate movements lead to stronger aggregate coordination and therefore to a higher procyclicality of the fraction of adjusters, which in turn increases the cyclical comovement of both the first moment of the investment rate distribution – from 0.3602 to 0.9321 – as well as the second moment, as can be seen in the following table. We thus confirm the conjecture in Kahn and Thomas (2005) that general equilibrium price movements are important to quantitatively account for cross-sectional business cycle dynamics.

Table 13: CYCLICALITY OF INVESTMENT DISPERSION AND GENERAL EQUILIBRIUM

Cross-sectional Moment	Baseline - GE	PE
$\sigma(\frac{I_{i,t}}{0.5*(K_{i,t}+K_{i,t+1})})$	0.6517	0.3134
Fraction of Adjusters	0.6413	0.4736

In summary, we finish this section by reiterating that both the extent of the procyclicality of investment dispersion as well as the countercyclicality of the dispersion of firm-level Solow residual innovations, and thus more generally cross-sectional business cycle dynamics, impose important and very tight restrictions on important structural parameters, such as adjustment frictions and factor elasticities in the production function. This makes the study of cross-sectional business cycle dynamics important for heterogenous-firm models. We also confirm the conjecture in Kahn and Thomas (2005) that general equilibrium price movements are important to quantitatively account for the cross-sectional business cycle dynamics observed in the data.

[26] In a monopolistic competition framework, column two implies a scenario with a CRTS-one-third-two-third production function and a markup of 1.25, column three a markup of 1.20. In each case, we recompute firm-level and aggregate Solow residuals and re-calibrate the adjustment cost parameter $\bar{\xi}$ to minimize $\Psi(\bar{\xi})$ in (12).

5.2 Robustness

In the following Table 14 we document robustness of our baseline result to some of the parameter choices we have made in the baseline calibration. We change one parameter at a time, but do not re-calibrate $\bar{\xi}$.

Table 14: PROCYCLICALITY OF INVESTMENT DISPERSION - ROBUSTNESS

Scenario	$\rho(\sigma(\frac{I_{i,t}}{0.5*(K_{i,t}+K_{i,t+1})}), HP(100)-Y)$
Baseline	*0.6517*
No excess kurtosis	0.5755
Higher volatility of $\sigma(\Delta\epsilon_{i,t})$	0.1368
Lower $\bar{\sigma}(\epsilon)$	0.9282
Lower volatility of z_t	0.4085
$CRRA = 3$	0.5855
Timing of $\sigma(\Delta\epsilon_{i,t})$	0.5182

In the second row, we check whether the introduction of a firm-level process for Solow residual innovations with quantitatively realistic excess kurtosis drives our result and the answer is negative. In order to check robustness of our results to a potential underestimation of the volatility of the countercyclical second-moment shock, we double it, while keeping its steady state value fixed at $\bar{\sigma}(\epsilon) = 0.1201$. We implement this by doubling the deviations from a linear trend in the $\sigma(\epsilon)$-process and re-estimating the unrestricted bivariate VAR between it and the linearly detrended aggregate Solow residual.[27] As expected, in this case the ability of the procyclical extensive margin effect to overcome the countercyclical second-moment shocks is limited, because the latter fluctuates more. This drives down the correlation of the investment rate dispersion and the cyclical component of aggregate output to 0.1368. Notice, however, that it is still positive, non-convexities in capital adjustment still cause a procyclical extensive margin effect that partially offsets the countercyclical second-moment shocks. But it is also clear from this exercise that the strongly procyclical investment dispersion that we find in the data – 0.613 – is at odds with the very volatile countercyclical second-moment shocks proposed in Bloom (2008) and Bloom et al. (2009) as important drivers of the business cycle. Halving the steady state $\bar{\sigma}(\epsilon)$ – see the fourth row –, in contrast, improves ceteris paribus the ability of the model to generate procyclical investment dispersion. This scenario is relevant, if one were to attribute some part of the measured $\bar{\sigma}(\epsilon)$ to measurement error in firm-level Solow residuals. The fifth row displays a scenario, where we lower the volatility of the first-moment shock so that the model now matches the volatility of the cyclical component of output, which in the base-

[27] The unconditional time-series percentage standard deviation of $\sigma(\epsilon)$ is 2.68% in the baseline case. We double that.

line calibration is too high in the model (see Appendix C for a discussion). This is effectively a lowering of the relative importance of first-moment shocks versus second-moment shocks, and it is important to make sure that our result is not driven by measurement error and too high a volatility in the aggregate Solow residual. Table 14 shows that this is not the case with the correlation of investment dispersion and the cyclical component of aggregate output being 0.4085. Nevertheless, the procyclicality of investment dispersion is reduced, as second-moment shocks have effectively become more important. Next, we checked whether our unity CRRA is driving our result by increasing the CRRA to 3. This results in hardly any change.[28] Finally, we checked whether the result is sensitive to the timing assumption about the revelation of the dispersion of the firm-level Solow residual innovation. The baseline model assumes that $\sigma(\Delta\epsilon_{i,t})$ is revealed today, concomitantly with z_t and ϵ_t, aggregate and idiosyncratic technology, and that both z_t and $\sigma(\Delta\epsilon_{i,t})$ predict the dispersion of the firm-level Solow residual innovation tomorrow through persistence in the VAR; there is another plausible timing assumption: $\sigma(\Delta\epsilon_{i,t+1})$ is revealed today. As the last row shows, this lowers somewhat the procyclicality of investment dispersion, but the extensive margin effect is still sizeable, as the corresponding number from a frictionless model would be -0.5432, compared to the -0.3845 in the frictionless counterpart of the baseline timing assumption.

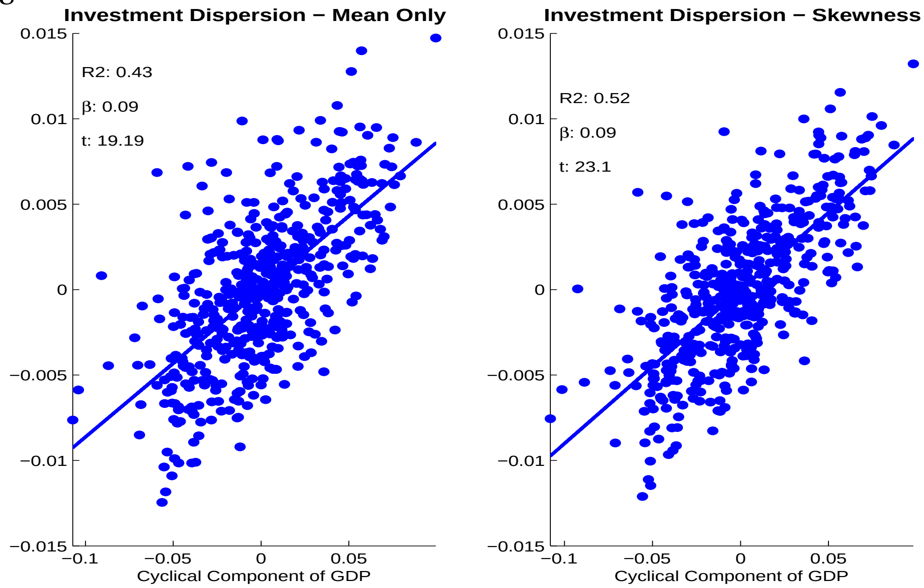
It remains to be shown that our result is not driven by the choice of only the average capital stock in the Krusell and Smith rules (10a) and (10b). While it is the case that in the presence of countercyclical second-moment shocks the conventional R^2-measure is fairly low – at least in some combinations of the discrete aggregate states –, and while it is also true that including the skewness of the capital distribution[29] leads to an average increase of the R^2 for the capital regressions from 0.9267 to 0.9925 and for the marginal utility of consumption regressions from 0.9974 to 0.9998, neither the aggregate behavior (see Bachmann and Bayer (2009) for details) nor the cross-sectional dynamics of the model are significantly altered: the correlation between investment dispersion and cyclical aggregate output raises slightly from 0.6517 to 0.7187. That means, if anything, our baseline numerical specification is somewhat conservative with respect to our main finding. The bottom line, however, is that better forecasts do not necessarily induce the agents to behave differently (see Bachmann et al. (2008) for a similar finding).

The scatter plots in Figure 3 make this point graphically: the positive relationship between investment dispersion and cyclical aggregate output is nearly indistinguishable between a numerical specification where only average capital is used as a state variable and one, where also the skewness of firm-level capital is included in the forecasting rules.

[28]Technically, with the separable felicity specification in (7) there is no balanced growth path with CRRA=3. The model remains consistent with balanced growth, if the disutility of leisure grows with the steady state growth rate, γ, and the fundamental discount rate is accordingly adjusted.

[29]Including the standard deviation of capital did not yield any interesting improvements in R^2. The average R^2 over all discrete states for the skewness regression, that is analogous to (10a), is 0.9703.

Figure 3: Cross-sectional Dispersion of Firm-Level Investment Rates and Solow Residual Innovations: Higher Moments



6 Final Remarks

This paper, to the best of our knowledge, is the first to study the cyclical behavior of the second moments of the cross-sections of firm-level innovations to value added, Solow residuals, capital and employment. We show that even in the presence of countercyclically disperse Solow residual innovations the dispersion of investment rates is significantly and robustly procyclical. We also show that this can be quantitatively explained by realistically calibrated non-convex adjustment costs: a procyclical extensive margin effect dominates the countercyclical dispersion in the driving force. Other potential explanations, such as financial frictions or selection effects, are ruled out. We finally argue that the understanding of the cross-sectional business cycle dynamics imposes important restrictions on structural parameters and driving forces. In particular, large countercyclical second moment shocks that could generate sizeable business cycle dynamics would be incompatible with procyclical investment dispersion.

We view this as just the beginning of a new research program that attempts to understand more comprehensively the time-series behavior of the entire cross-section of firms, not merely the cyclicality of second moments. This will ultimately lead to a better microfoundation of structural heterogeneous-firm models and contribute to making them suitable for policy analysis. We also plan to corroborate these new findings for more countries, in particular the U.S.

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A Appendix A - Data Appendix

TO BE COMPLETED

B Appendix B - Robustness of Cross-sectional Cyclicality

In this appendix we check the robustness of the main finding of this paper – the procyclicality of investment dispersion – to sample selection and variable construction. First, we use an aggregate price deflator for investment goods (specifically: *Preisindex der Investitionsgüterproduzenten*, source: *Statistisches Bundesamt*) in the perpetual inventory method instead of sectoral deflators separately for equipment and structures. Secondly, we employ a stricter outlier removal criterion of 2.5 standard deviations around the year-specific mean in Solow residual and value added innovations, as well as investment rates and employment changes. Thirdly, we use a more liberal outlier criterion using 5 standard deviations instead of 3. Fourthly, we employ a specification, where we assume that an outlier above 3 standard deviations means a merger and, subsequently, treat these firms as new firms in addition to removing them in the year, where the outlier occurs. Fifthly, we restrict the sample to those firms with at least 20 observations in first differences, in order to make sure that the cyclical effects we find are not due to cyclical variations in the sample composition. Sixthly, we use all the firms that we observe at least twice with first differences. Finally, we carry out a more standard perpetual inventory method that uses the reported capital stocks in the first year of observation for a firm, instead of solving a fixed point problem in capital prices (see Appendix A for details). As one can see from Table 15 the results are robust to all these alternative sampling procedures.

Table 15: CYCLICALITY OF CROSS-SECTIONAL INVESTMENT DISPERSION - DATA TREATMENT

Treatment	$\rho(\sigma(\frac{I_{i,t}}{0.5*(K_{i,t}+K_{i,t+1})}), HP(100)-Y)$
Baseline	0.613
Uniform price index for investment goods	0.637
Stricter outlier removal	0.606
Looser outlier removal	0.549
Stricter Merger Criterion	0.617
Longer in sample	0.568
Shorter in sample	0.624
Standard Perpetual Inventory	0.630

C Appendix C - Aggregate Statistics

Table 16: AGGREGATE BUSINESS CYCLE STATISTICS FOR THE BASELINE CALIBRATION

Moment/Aggregate Quantity	Y	C	I	E
Standard Deviation	3.37% (2.30%)	1.24% (1.78%)	15.30% (4.37%)	2.47% (1.80%)
Relative Standard Deviation	1	0.37 (0.77)	4.55 (1.90)	0.73 (0.78)
Persistence	0.30 (0.48)	0.62 (0.68)	0.23 (0.42)	0.21 (0.61)
Correlation with Y	1	0.80 (0.67)	0.97 (0.83)	0.95 (0.68)

All variables are logged and then HP-filtered with a smoothing parameter of 100. The numbers in brackets are the statistics from the data, from the sectoral aggregate that corresponds to the USTAN data: the non-financial private business sector. They are gathered from German sectoral national accounting data. Real private consumption data are taken from table 3.2 in the *Volkswirtschaftliche Gesamtrechnungen*, source: *Statistisches Bundesamt*. The model employment variable includes the workers that adjust the firms' capital stocks.

In our baseline calibration, the economy is overall too volatile, which we attribute partly to the fact that we compute the aggregate Solow residual process from the private non-financial business sector and not from the overall economy. It is also partly due to the existence of a second aggregate shock, the second-moment shock: for instance the volatility of output drops slightly to 3.18% and that of investment to 13.92%. And it is finally due to the relatively coarse aggregate grid, $[5,5]$, which generates some artificial volatility that is diminished with a grid of $[9,9]$. Since this grid choice does not affect our baseline cross-sectional result and increasing the aggregate grid size from $[5,5]$ to $[9,9]$ is an enormous computational burden, we report the baseline and the robustness results with the coarser grid.

Nevertheless, both the too high volatility numbers, as well as the too low persistence numbers as well as the discrepancy between model and the data in the relative standard deviations – relative to $std(Y)$ – of aggregate consumption and aggregate investment show that there is not enough smoothing in the baseline calibration, which is a standard problem of the standard RBC model. Our baseline model cannot improve that, as the level of non-convexities essentially puts it in a parameter range, where the Kahn and Thomas neutrality result still holds (see Kahn and Thomas, 2008). Since this paper is exclusively concerned with cross-sectional dynamics, for which – as we have shown – non-convexities matter already at a level, where they would be near-neutral for aggregate dynamics, we do not view this as a problem for our main result. More smoothing could be implemented through a standard quadratic adjustment cost element on top of the fixed cost, however at both a substantial computational burden and at the expense of cleanness of exposition. In fact, quadratic adjustment costs would work very similarly to an increase in curvature in the maximized-out revenue function, which, as we have shown, puts more emphasis on the procyclical extensive margin and will only strengthen our mechanism. Our robustness checks include a case, where we decrease the volatility of the aggregate Solow residual in order to match the volatility of aggregate output. This is the most conservative scenario, as this puts relatively more weight on the second-moment shocks, i.e. the countercyclicality of the dispersion in the Solow residual innovations, and would make – it all things equal – harder for the extensive margin effect in the lumpy model to generate procyclicality of investment dispersion. Row five in 14 in Section 5.2 shows that this hardly changes our baseline result. To summarize: the aggregate shortcomings of the model are similar to the one in the standard RBC model, but based on our robustness checks we view them as mainly orthogonal to the cross-sectional dynamics that this paper focusses on.