1) Two manufacturing firms, A and B, create pollution as a byproduct of their production. Firm A’s pollution abatement costs are $x^2+50$, and firm B’s abatement costs are $2x^2+100$. The benefit to society from one unit of pollution abatement is constant and equal to $100$.

   a. What is the socially optimal level of pollution abatement? What is the socially optimal allocation of pollution abatement across firms?
   b. What subsidy per unit of pollution abatement will result in the socially optimal outcome?
   c. Suppose the government decides to issue tradable pollution permits. Each permit allows the firm to produce one unit of pollution. Suppose they issue enough permits so that each firm, if it is unable to trade, will have to reduce pollution by 45 units. What is the market-clearing price of permits (assuming perfect competition in the permit market)? What level of abatement does each firm engage in? Is this socially optimal? Why or why not?
   d. How many more permits should the government issue to reach the socially optimal level of pollution abatement?

**Suggested answer:**

1. a. The marginal costs of pollution abatement are $MC_A = 2x$ and $MC_B = 4x$. The socially optimal level of pollution abatement for each firm is where the marginal cost is equal to the marginal benefit. For firm A this is $2x = 100$ so $x = 50$. For firm B, $4x = 100$ means $x = 25$. So the socially optimal level of pollution abatement is $50 + 25 = 75$. Firm A should reduce pollution by 50 units, and firm B should reduce pollution by 25 units.

   b. The subsidy that will result in the socially optimal outcome is the social marginal benefit, i.e. 100.

   c. We have two constraints. First, the total amount of abatement must equal 90 units. Second, for the market to be in equilibrium, the marginal cost of abatement must be equal across firms. So we have $x_A + x_B = 90$ and $2x_A = 4x_B$. Solving this system yields $x_B = 30$, $x_A = 60$. The market-clearing price will equal the marginal costs, so $P = 120$. This level of abatement is not optimal, because there are too few permits available.

   d. Currently it is requiring 90 units of abatement. The optimal level is 75. So the government should optimally issue 15 more permits.

2) For each of the three examples below, answer the following questions

   i) Is there an externality? If there is, is the externality positive or negative? Is it a consumption or a production externality?
   ii) If there is an externality, is it likely that private markets will develop that will allow the externality to be internalized?
   iii) If there is a externality, contrast the effectiveness of 1) direct quantity regulation, 2) taxes/subsidies, and 3) permits with trading as a way of optimally correcting the externality.
a) Subsisting on a diet of fast food
b) Talking on a cell phone in the library
c) Getting a high school education.

Suggested answer: A consumption externality results where an individual’s private consumption of a good causes another to experience disutility. A production externality results where something about the production process of a good causes another to experience some disutility. In the cell phone example, excess noise is cause by the consumption of cell phone minutes.

Be clear about “private market solutions.” The way the private market would generally settle externalities is by paying the one who creates the externality to reduce their externality-generating activity (or by coercing them to stop). In the education example, the existence of private schools is not a private solution to education externalities. Private schools allow parents to purchase higher quality education if they think the benefits exceed the tuition costs. Private schools do nothing to help anyone internalize the externality of education. A “solution” to the externality would be anything that encourages those who don’t receive a high school education to do so. So one private market solution would be for people to band together and pay dropouts to remain in school.

a) This could be a negative consumption externality because of the higher health care costs associated with obesity. They key issues are 1) whether a fast food only diet would cause obesity, 2) whether the obese have higher health care costs, and 3) whether the health premiums reflect the higher expected costs. My guess is that there would, in fact, be a negative consumption externality (though there perhaps could be some Medicare and Social Security saving).

Quantity regulation is likely to be difficult. A weight tax or a skinny subsidy might be feasible (though politically difficult) as could a “fast food” tax. Permits giving (each American) the right to eat a certain amount of fast food would be administratively cumbersome. If one were truly worried about this issue, a tax approach would probably work best.

5b) This is a negative consumption externality (it’s not a production externality because the cell phone users are not using the phone to produce anything of value to others). A private market solution would exist if one or more library users paid the offending cell phone user. Physical threats or social norms might also solve the problem. Quantity regulation (no cell phone use) is probably the best and most efficient solution. Taxes would be an additional hassle for librarians.

5c) I wanted you to think of this as a positive consumption externality (from the perspective of individuals). But if you described the problem clearly, you could take the perspective of a educator and describe schools as creating a positive production externality. The positive externality could come from the fact that education is positively correlated with voting, not committing crimes, health, and perhaps there are “peer effects” so that well educated people make those around them better.

There is not an obvious private solution to correct the externality. Quantity regulation (mandating school attendance up to certain ages) or subsidies (making school free) are what we
currently do to address the problem. Permits, raising the costs of dropping out by requiring dropouts to pay a “dropout tax” would be an “out of the box” solution.

3) Suppose Paul and Scott are the only residents of a city (others may pass through and possibly commit crimes). Each has a utility function over cigarettes (X) and total policemen (M), of the form: \( U = 2 \log(x) + \log(M) \). The total policemen hired, \( M \), is the sum of the number hired by each person, so \( M = M_p + M_s \).

Paul and Scott both have income of 100, and the price of both cigarettes and a policeman is 1. They are limited, for the purposes of this problem, to providing between 0 and 100 policemen.

a) How many policemen are hired if the government does not intervene? How many are paid for by Paul? By Scott?

b) What is the socially optimal number of policemen? If you answer differs from a), why?

c) Suppose the broader county government is not happy with the private equilibrium, and it decides to provide 10 policemen for the town. It taxes Paul and Scott equally to pay for the new hires, but each is free to hire additional policemen if they would like. What is the new total number of policemen? How does your answer compare to a)? Have we achieved the social optimum? Why or why not?


Max \( 2 \log(x_p) + \log(M_p + M_s) \) such that \( 100 = x_p + M_p \). Use the budget constraint to substitute for \( x_p \). Maximize with respect to \( M_p \) and set equal to zero. You’ll find \( M_p = \frac{100}{3} - \frac{2}{3} M_s \).

Scott, who faces a symmetric problem, has a similar first order condition (also called a reaction function): \( M_s = \frac{100}{3} - \frac{2}{3} M_p \). Solve the two reaction functions and find that \( M_p = 20 = M_s \), so the total number of policemen is 40.

b) Social optimum occurs when the sum of the marginal rates of substitution equals the marginal rate of transformation. The \( \frac{P_M}{P_x} = 1 \). The \( MRS_p = \frac{MU_M}{MU_x} = \frac{1}{2} \frac{x_p}{2(M_p + M_s)} \).

So the \( \sum MRS = \frac{x_p + x_s}{2M} = MRT = 1 \). Furthermore, we know the budget constraint looks like \( x_p + x_s + M = 200 \Rightarrow x_p + x_s = 200 - M \). Thus \( M = 66 \frac{2}{3} \). The level is greater than in (a) because when Paul or Scott considered adding another policeman, they just considered their individual benefit, ignoring the fact that an additional policeman bring a benefit to the other person. This is like an externality.

c) This problem is solved the same way you solved part a). The setup is

Max \( 2 \log(x_p) + \log(M_p + M_s + 10) \) such that \( 95 = x_p + M_p \). The best response function looks like \( M_p = 25 - \frac{2}{3} M_s \). Scott’s is the same. Solve these two, and you will find that
$M_p = M_s = 15$, which implies that $M = 40$. Since Paul and Scott are taxes equally and the amount provided is less than what they would provide without any government, they treat the tax as if they were just paying for five policemen on their own. Since they wanted 20, they supplement the 5 they pay for with their taxes by purchasing an additional 15. In this sense, the police the government provides fully crowds out the privately provided policemen.

4) Suppose that two individuals, Jon and David, form a community and would like to construct a communal fort that would protect them from attacks. They consume both good $X$, a private good, and the protection from the fort, $P$. One unit of good $X$ costs 1 unit while one unit of $P$ costs 2 units, so the budget constraint for each is given by: $X_i + 2P_i = 100$. Both Jon and David have an income of 100 and a utility function of the form:

$$U = \log(X_i) + 2\log(P_j + P_D)$$

a): How much protection, $P$, will be privately provided? What is optimal consumption of $X$, the private good?

b): What are the socially optimal amounts of protection, $P$, and consumption, $X$, of the private good? This is an example of what phenomenon discussed in class?

**Brief solution:** The problem is to maximize $U = \log(X_i) + 2\log(P_i + P_2)$ subject to $X_i + 2P_i = 100$.

Substitute from the budget constraint into utility and get $\log(100 - 2P_i) + 2\log(P_i + P_2)$ and differentiate with respect to $P_i$, finding $\frac{-2}{100 - 2P_i} + \frac{2}{P_i + P_2} = 0$. Manipulate and you will find the reaction function $3P_i = 100 - P_2$. The problem is symmetric, so the reaction function for David (Mr. 2) is $3P_2 = 100 - P_1$. Solve these and find $P_1 = P_2 = 25$, and $X_1 = X_2 = 50$. You can also find the reaction functions by calculating the MRS and setting it equal to the price ratio. Specifically, the MRS for Mr. 1 is $\frac{P_i + P_2}{2X_i}$ and the price ratio is $\frac{1}{2}$. This will give you the same reaction function as given above.

b. A straightforward way to solve for the social optimum is to maximize the sum of utility, subject to the budget constraint, so the problem would read, maximize $U = \log(X_i) + \log(X_2) + 4\log(P_i + P_2)$, subject to $X_i + X_2 + 2P = 200$, where $P_1 + P_2 = P$. Given the problem is symmetric, we also know $X_1 + X_2 = X$, and $X_1 = X_2 = \frac{1}{2}X$. So we can write the problem for the socially optimal solution, substituting in from the budget constraint, as

$$\max U = 2\log[100 - P] + 4\log(P) \Rightarrow \frac{-2}{100 - P} + \frac{4}{P} = 0.$$ This implies $P_1 = P_2 = X_1 = X_2 = 33\frac{1}{3}$. The phenomenon this problem illustrates is the general idea that public goods will be underprovided by the private market.

For those solving the problem by equating the sum of the marginal rates of substitution equal to the price ratio, your answer would note that $X_1 + X_2 = X$, and $X_1 = X_2 = \frac{1}{2}X$. Set up the problem as maximizing $U = 2\log(\frac{1}{2}X) + 4\log(P)$ subject to $X + 2P = 200$. The sum of the MRS is $\frac{MU_X}{MU_P} = \frac{2P}{4X} = MRT = \frac{1}{2}$. This, along with the budget constraint yields the previous answer.
5) Suppose that the demand for a chemical is given by \( Q = 100 - 2P \), where quantity is measured in pounds. The market supply is given by \( MC = 5 \). Assume that the marginal external damage of this product is \$3/unit.

a): What is the equilibrium price and quantity of this good without government intervention?

b): What is the socially optimal \( P \) and \( Q \)? What Pigouvian tax or subsidy would generate the socially efficient level of the good?

**Brief solution:** Solving \( P \) in terms of \( Q \) from the demand curve, we get that inverse demand is given by \( P = 50 - 0.5Q \). Setting that equal to the supply, we get \( 5 = 50 - 0.5Q \), which implies that \( Q = 90 \). Find the price by plugging 90 in for \( Q \) in the inverse demand, and we find that \( P = 5 \).

b): The social marginal cost curve is \$8. Then we find that \( 8 = 50 - 0.5Q \), which means that \( Q = 84 \) and \( P = \$8 \) is the socially optimal amount and price. That is, this the amount at which social marginal benefit equals social marginal cost. A Pigouvian tax on the producer of this product equal to \$3/pound for every pound it will result in this outcome.

6): Suppose that Mary and Beth live on the same street. In the winter, both of them like the snow on their street to be plowed. Beth's demand is given by \( Q = 40 - P \) and Mary's demand is given by \( Q = 30 - 2P \). Suppose that the marginal cost of plowing the snow is constant at \$35. What is the socially efficient amount of plowing that should be done?

**Brief solution:** Add the demands vertically. Solving for both inverse demand curves, we get \( P = 40 - Q \) and \( P = 15 - 0.5Q \). Adding those where prices are both positive, we find that the segment of the social marginal benefit curve from where \( 0 < Q < 30 \) is given by \( P = 55 - 1.5Q \). Mary is no longer willing to pay anything when \( Q \) exceeds 30, so there the social marginal benefit curve is \( P = 40 - Q \). Find the quantity where the social marginal benefit is equal to social marginal cost, which is \$35. So \( 35 = 55 - 1.5Q \rightarrow Q = 13.33 \). Since this is less than 30, we are indeed using the correct segment of the social marginal benefit curve.

7): Suppose a firm is considering an investment. The investment will cost \$250,000 and return \$60,000 per year for each of the next 5 years. Suppose the tax system allows straight-line depreciation with a 10-year asset life. The firm uses a 10 percent discount rate (that is, the opportunity cost of funds is 10 percent) and the corporate tax rate is 35 percent.

a) What is the present discounted value of the returns from the investment?

b) Should the firm make the investment?

**Brief Solution:**
The present value of the returns is \( 60000 + 60000/1.1 + 60000/1.1^2 + 60000/1.1^3 + 60000/1.1^4 = 250,192 \). The firm also gets tax benefits. They will be able to deduct 10% of the investment, or \$25000, every year for ten years. Since the tax rate is 35%, the value to the firm of \$1 in tax deductions is \$0.35. So the present value of the tax deductions is \( 0.35[25000 + 25000/1.1 + ... + 25000/1.1^9] = 168,976*0.35 = 59,142 \). The total present discounted value to the firm of making the investment is thus \$59,142 + 250192 = 309,334 \). This is greater than the cost of the investment (\$250000) so the firm should make the investment.