Editor’s Introduction

William Allen Brock. Fondly known by family, friends and colleagues alike as Buz. I first met Buz when I was a graduate student at Cornell University. At the urging of Ed Burton, then a member of the faculty, I introduced myself to Buz with, “I’m looking for a thesis topic.” With a characteristic smile on his face he asked, “What have you studied?” I hesitantly answered, “Electrical Engineering, Applied Mathematics . . .” He broke in, “Sit down. I’ve got just the thing for you.” Pulling out a yellow tablet and a box of Ticonderoga #2 pencils, he launched my career in economics.

Buz’s enthusiasm for the subject is unbounded. He is fascinated by the forces of economics and their impact on human behavior. Eschewing static models from the very beginning of his career, Buz has turned to dynamic models of intertemporal economics as the primary tools for his research. Although his thesis deals with technical issues in the infinite horizon growth model, his research was soon to show his strong feelings and commitment to economics as a social science. Interspersed with the papers in which he developed the tools of dynamic economic analysis, are such papers as “Money and Growth” (1974a) and “The Polluted Golden Age” (1977). Then, at the end of the decade and into the eighties, he produced a number of articles and books on various aspects of regulation and the impact of government policy on individual welfare. Much of this research was carried out jointly with David Evans, Stephen Magee and Stephen Turnovsky.

Advisor, coauthor, colleague, friend. I have had the benefit of nearly a quarter century tutorial with Buz. Three times I held a visiting position at the University of Wisconsin, and I spent many summers in Madison working with Buz. At lunch, or in a coffee house on State Street, Buz would show me his latest results and sketch out his ideas on how to make progress on a problem. Often our conversations would turn to the social implications of economic policy. Buz is a complex man, inside of whom rages a battle between his conservative and progressive roots. He loathes and despises the way common politicians use the government, and at the same time longs for the social benefits of a benign government. And so he seeks an economic rather than a political solution to current problems. His recent research on the environment exemplifies this direction of his research. But deep at heart,

\footnote{Almost from the beginning. By the time Buz went to graduate school at the University of California at Berkeley, he had already published three articles. His study of dynamics began at Berkeley.}
he knows that economics and politics are inexorably intertwined.

In evaluating Buz’s research there are two things that stand out: the richness and the depth of his contributions to the theory of optimal growth; and the breadth of the applications of the intertemporal model to economic issues. Nevertheless, in spite of the success he enjoyed with the optimal growth model, he also has written in a wide variety of areas. His contributions to the theory of nonlinear dynamics are numerous, and although his current research in Limnology with Carpenter and Ludwig (1998) takes him back to the optimal control model, his work in the analysis of complex dynamics in economics is far from over.

Selecting the articles to publish here was not easy. The ones that do appear here reflect some of Buz’s best work and are indicative both of the type of problem that he finds challenging and the methodology that he uses to solve them. Some of the articles were also chosen to bring into one volume the many and disparate facets of his research career. Also included in this volume are Buz’s own introductions to his work, which help to tie together the main thrusts of his research to date.

Often in the morning I find my mailbox stuffed with Buz’s notes from the night before. His practice is to take a short nap before dinner, thereby “getting two work days out of one.” A true scholar, Buz meets the challenge to find the answer. It is in the science that he derives his intellectual satisfaction.
Brock’s Introduction and Discussion

Dedication

I thank my wife, Joan, and my daughter, Caroline, for their key roles in making my life worthwhile and for their support of my science. I also thank Lionel McKenzie and Blanche McKenzie for their treasured friendship and for their contributions to my life.

Acknowledgments

I was raised in a family where my father worked as a union electrician in order to support the family farm. Growing up in agriculture gives one a first hand experience with the competitive economic process. This was economics in action. I decided I wanted to try to understand these forces that so dominated our lives on the farm.

My initial introduction to academic economics was working for the agricultural economist, Russell G. Thompson, as a research assistant at the University of Missouri, Columbia during the early sixties. Russell taught me basic economics, econometrics, and an appreciation for empirical research in economics.

When it was time to apply to graduate school Russell advised me to study mathematics because he said, it was easier to learn and master mathematics while “young.” He said it was easier to study economics when “older,” because good economics required wisdom and maturity. Russell also argued that my mathematical training would allow me to keep my focus on substantive economic research.

I still think that’s the best advice I ever got from anyone. Russell was a great mentor and had a large influence on me. During that period, I would also talk with Professor Oscar Burt who also served as a fine mentor who tantalized me with the beauty of recursive dynamical arguments, at which he is a master.

Another big influence on my research style came from being a graduate student in mathematics at the University of California at Berkeley, where one of the first things we learned how to do was to construct counterexamples. This type of training sets in one’s mind the power and limitations of a set of assumptions (or axioms).

This mathematical training plus the goal of generating econometrically
tractable theoretical structures that could be readily adapted to real applications and substantive economic issues tilted my research more towards the dynamical systems style of modeling than towards the abstract general equilibrium style of modeling.

My first interest in mathematics was dynamical systems theory. Berkeley was the place to be in that field at the time. When I found out that David Gale was at Berkeley I took a reading course from him and became interested in optimal growth theory. David was not only a great teacher and mentor, he also became a dear friend and co-author as well. I went to Copenhagen to be with him in the fall of 1968, which is where we wrote our joint article.

I arrived on a snowy day in the winter of 1969 for my first academic job at the University of Rochester, Rochester, New York. I was very lucky to start my academic career there because I gained a wonderful friend and mentor, Lionel McKenzie, as well as meeting another dear friend and co-author, Leonard Mirman. The Economics Department at Rochester was a perfect environment to launch the career of a young scientist. Rochester mentored me and sheltered me from the kinds of nonsense that destroy so many young scholars. Mirman and I wrote our two joint papers during my time at Rochester.

I moved to the University of Chicago in 1972 and fell in love with the seriousness with which economics was taken at Chicago, as well as its price theoretic and empirical bent. Chicago’s desire to understand that marvelous complicated machine called the economy was infectious and I have been infected ever since.

I loved the intellectual excitement of Chicago. But, for two people, such as my wife and me who were reared in a country setting, the desire to situate our family in a more rural environment for the rearing of our daughter, Caroline, finally resolved itself in trial part–time moves to Cornell University and the University of Wisconsin, Madison. We ultimately ended up at Madison. I finally gave up Chicago in the early eighties and I still miss it.

Madison has provided me with the optimal tradeoff between quality of living environment and quality of intellectual life. The department at Madison has nurtured my work and has provided many great colleagues. The University has supported my research through the Wisconsin Alumni Research Foundation and the Graduate School. The award of a Vilas Research Professorship in 1990 has been key to my work since 1990. I also wish to acknowledge the key roles that the National Science Foundation and the Vilas Trust have played in my life. It simply is not possible (especially in today’s environ-
ment) to do decent science without support like this. I am deeply grateful for it.

Many people have been key to my career. Bob Lucas and Tom Sargent have played the roles of friend and mentor for more years than all of us like to admit. Dee Dechert, Steve Magee, and José Scheinkman are best friends and co-authors. Life without them would be boring. I especially thank Dee Dechert for the interest he has shown in my work as well as the encouragement he has given me as exemplified, for example, by his taking the time to edit this volume of essays.

Introduction

This book of selected essays is divided into four sections. The section headings are given below together with the list of papers in each section. The papers are unified for the most part by the desire to create analytically and econometrically tractable models of “toy economies” that teach us lessons about the role that economic forces play in shaping the patterns in the economic data that we see.

Ever since working for Russell Thompson as an undergraduate, I have been fascinated by the patterns seen in economic data and obsessed with trying to understand the forces that shape these patterns. Much of the model building and analytical model exploration presented below can be seen as part of this general effort to understand patterns in economic data.
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Section A: Stochastic Models

The papers reprinted in this section are: 1. Brock and Mirman (1972); 2. Brock and Majumdar (1978); and 3. Brock and Magill (1979). These papers extend the basic turnpike results of deterministic optimal growth models to the stochastic case.

The first paper extends the turnpike results of the one sector neoclassical growth model to the stochastic one sector growth model with productivity shocks. The second two papers develop turnpike results for discrete time multisector and continuous time multisector cases respectively. I discuss the evolution and applications of these papers below.

While conducting my thesis research, Brock (1970), written under the supervision of David Gale at Berkeley, I was instilled with a long term fascination with the study of infinite horizon optimal planning models. In my thesis I studied infinite horizon deterministic optimal planning problems under zero discounting of the future. I extended previous work in this area by replacing the notion of “overtaking optimal” with “weakly maximal,” by creating an “auxiliary” functional which was always non negative over all planning trajectories. Using this functional along with standard tools of analysis such as Fatou’s Lemma, I was able to get a straightforward proof of existence of a weakly maximal solution.

Besides existence, in my thesis I also proved the convergence of all weakly optimal paths to a unique steady state. This property is called the “Global Asymptotic Stability Property” abbreviated as “GAS.” This work represented a modification of existing “value loss” methods in a form that made extensions to stochastic settings easier (See, for example, Arkin and Evstigneev (1987).) See Carlson, Haurie, and Leizarowitz (1991) for an up-to-date treatment of the theory in continuous time settings.

Given this work it was natural to attempt to carry out the same research program for stochastic settings. This attempt lead to two papers with Mirman (1972), (1973); the paper with Majumdar (1978), and the paper with Brock and Magill (1979)Magill (1979). A simple statement of the results of this work is that we got the same type of results in the stochastic case as in the deterministic case by replacing points with distributions. I.e., the optimal trajectory in the stochastic case is a sequence of random variables, and the analog of the GAS result is that there is a unique distribution such that the optimum sequence of random variables converges to this distribution independently of initial conditions just like the deterministic case.
While this is true at an abstract mathematical level, there are substantive differences in the economics between the deterministic cases and the stochastic cases. I pointed out some of these substantive differences in a comment on Roy Radner’s 1972 work in Brock (1974a). First, I showed that one could make a representative agent rational expectations model out of the Brock and Mirman (1972) stochastic growth model and analyze different equilibrium issues with this kind of model. For example, I discussed the pricing of financial assets with this device in Brock (1982). See Becker and Boyd (1997) for a discussion of these types of “Equivalence Principles” where one can turn optimal planning models into rational expectations dynamical general equilibrium models and use the results for business cycle and financial analysis.

The Brock and Mirman work served as a building block for the type of business cycle work (especially real business cycle work) which is discussed in Cooley (1995), Sargent (1987), and Lucas (1987). This was useful because the Brock and Mirman paper could be turned into a recursive stochastic dynamic general equilibrium model with rational expectations. The ergodicity of the intertemporal equilibrium created a fruitful framework for hypothesis testing because time series econometric practice is aided by ergodicity and stationarity.

The original Brock and Mirman paper reprinted here contains two basic results. The first one shows that concavity of the reward function and the production function imply that the optimal stochastic process has only one ergodic class. This result is the analog of the deterministic case where concavity of reward and production implies there is only one optimal steady state. The second part shows convergence in distribution to a “steady state” stochastic process.

Given the first result, we could have used standard results which were in Mirman (1973) to get the second result. However, we wished to include an analytic, self contained, proof of the second result. Unfortunately that proof contained a gap as reported by Bassanini (1996). He repaired this gap and users can go on using the second part of the Brock and Mirman paper as before. The first major result of the Brock and Mirman paper is not effected by Bassanini’s finding.

Secondly, in an attempt to show the dramatic differences in economic substance between deterministic and stochastic theories, I pointed out that a consequence of the introduction of uncertainty was the breakdown of the labor theory of value. In the one sector case this breakdown shows up in the
stochastic steady state dependence upon elements of the utility function, not just the discount factor on the future as in the deterministic case.

In Brock (1974b), I showed how nonsubstitution theorems could be used to study optimal steady states in deterministic multisectoral infinite horizon optimal growth models under conditions of no joint production, one primary factor of production, and constant returns to scale in production. It turned out that relative prices in optimal steady states only depend on the discount rate on the future and not on features of the utility function. This is a form of the labor theory of value. I.e., relative prices in the optimal steady state depend only upon “congealed labor” coefficients and the discount rate on the future, not upon features of demand, i.e., the utility function. This neat separation breaks down in the stochastic case. The essential reason is that the stochastic case generates joint production of outputs in different states of the world and this jointness induces features of the utility function into quantities such as relative prices in the stochastic steady state.

Third, there are many other new issues raised by the presence of uncertainty such as asset pricing, testing for ergodicity, and option values induced by possible hysteretic effects in systems with alternative stochastic steady states. Furthermore fields such as finance, macroeconomics, growth theory, and public finance have used stochastic optimal planning models as building blocks.

Fourth, the papers with Magill and with Majumdar attempt to extend the single sector theory to multisectors. It is well known that there can exist multiple steady states for multisector models in deterministic settings so we should not expect matters to be better in multisector stochastic settings. Nevertheless, for the multisector case with concave reward and production functions it was shown by several authors (including Brock and Scheinkman (1976)) during the mid seventies (cf. McKenzie (1986)) that if the discount rate on the future was small enough then there was a unique optimal steady state and all optimal paths converged to it. In the papers with Magill and with Majumdar, we formulated a stochastic analog of this theory and proved convergence theorems. Perhaps these papers may have a similar use in a future, more disaggregated macroeconomics as did the Brock and Mirman model in aggregative macroeconomics. Arkin and Evstigneev (1987) and Marimon (1989) contain nice statements of the state of the art in multisector stochastic theory in the Russian and Western literatures.
Section B: Financial Modeling and Related Macroeconomic Modeling

In this section, we have reprinted four papers: 1. Brock (1982); 2. Brock and Magee (1982); 3. Brock and LeBaron (1990); and 4. Brock and LeBaron (1996). These papers represent a sample of our own applications of the theories discussed in Section A to substantive economic problems.

The first paper, Brock (1982) represents an extension of Lucas’s exchange economy theory to production based asset pricing models. This extension opens up avenues to many applications which are not possible in a pure exchange setting with no production activities. While this paper has been used for many applications, it has been hampered by the lack of analytical tractability and the presence of a very limited class of closed form solutions. However important advances in numerical solution methods (c.f., Judd (1998)) have opened up the possibility of using this kind of model in a large set of potential applications. For example, Akdeniz (1998) and Akdeniz and Dechert (1997) have used numerical solutions of the model to uncover equilibrail economic forces that shift the static Capital Asset Pricing Model’s efficient frontier over the business cycle.

Brock and LeBaron (1990) used the model to illustrate the impact of financing constraints on a subclass of firms in the model to uncover understanding of documented empirical findings of differential degrees of mean reversion across firm sizes and across episodes of liquidity stress such as the Great Depression. They argued that financially constrained firms should display greater mean reversion than unconstrained firms. There appears to be empirical evidence consistent with this argument.

For example, in a panel study of mean reversion and finance constraints, Jog and Schaller (1994, p. 206) find “Our results are consistent with the hypothesis that finance constraints accentuate mean reversion.” They further state, “Our results suggest that production-based asset pricing models may be a fruitful area for theorists, providing insight into empirical regularities which have not been fully explained in terms of consumption-based models.” (Jog and Schaller (1994, p. 207).)

The second paper shows how the Brock (1982) model and the deterministic theory of macroeconomic effects of tax distortions of Brock and Turnovsky (1981) can be extended to the stochastic case. The theory is then extended to develop a theory of the required rate of return for public utilities when there are distorting taxes both at the corporate level and the personal level.
This theory leads to tractable formulae for the equilibrium value of claims such as stocks and bonds on firms in the presence of corporate and personal taxes. The formulae also reveal differences in the required rate of return for public utilities located in procyclical markets versus a-cyclical and counter cyclical markets. The formulae also show how some taxes such as a pure corporate income tax with perfect risk sharing by the government can be more distortionary than thought at first blush when general equilibrium effects are taken into account.

The general equilibrium effects of taxes in models like those of Brock and Turnovsky (1981) and Brock (1982) have been recently discussed in, for example, Altug and Labadie (1994). They show how the model not only can be used for a general theory of macroeconomic effects of tax distortions, but also it can be used to build a theory of the dynamics of Tobin’s Q values of business firms. See Turnovsky (1995) for much more on this topic.

However, even though this model has shown its usefulness in applications, we do not wish to overstate the ability of a model like Brock (1982) to fit the stylized facts. For example, Rouwenhorst (1995, p. 322) states in describing results from a version of the Brock (1982) model and the Brock and Mirman (1972) model: “Despite the model’s relative success reported in previous literature in describing certain key features of business cycles, its asset pricing implications were often at variance with the stylized facts. Nevertheless the model provided important clues about elements that equilibrium models must incorporate for explaining the behavior of returns over the business cycle.” He goes on to review the inability of this type of model to replicate bond and stock returns differences (called the “Equity Premium Puzzle,”) and how the introduction of production can make this problem even worse. He points out some other conflicts between this type of model and the data.

There are several routes towards fixing the problems that the neoclassical type of stochastic growth models have in matching the data. A promising route is to introduce “imperfectly competitive product markets...At most a small increase in the state space is required....The degree to which this direction of generalization of standard models can improve the ability of such models to explain observed aggregate fluctuations is the subject of continuing research.” (Rotemberg and Woodford (1995, p. 290)).

In later work Rotemberg and Woodford (1996, p. 87) state “We have demonstrated that the forecastable movements in output, consumption, and hours — what we would argue is the essence of the “business cycle” — are inconsistent with a standard growth model disturbed solely by random shocks
to the rate of technical progress.” They give a list of modifications that might fix the problems which include inventory dynamics, slow adjustment of the labor force as assumed, for example, in models of labor hoarding, and sluggish price and wage adjustments. For example, they state “one expects contractions in aggregate demand to reduce output, consumption, and hours when prices or wages are rigid. One would thus anticipate that all three series rise together in the aftermath of a negative shock to aggregate demand.”

The bottom line appears to be this: Basic stochastic growth models can serve as building blocks but they must be modified in several directions to be consistent with the data. For example, Black (1995, p. 159) states during a review of the ability of various models to explain facts that “Brock (1982) has a multi-sector stochastic growth model that generalizes the single-sector model of Brock and Mirman (1972, 1973). If we add nonseparable utility, adjustment costs for moving capital from one sector to another, human capital, and a few other features, we will have a model of the kind I favor.”

Lucas (1987) argues that the kind of dialogue between model building and data illustrated above where one confronts an analytically and empirically tractable model containing strong economic forces with the data, represents economic science at its best. I also believe that this kind of dialogue helps us understand the role that equilibrium economic forces play in shaping the patterns we see in the data.

Section C: Ecology, Mechanism Design, Regulation


Brock (1977) represents an early attempt to introduce the costs of economic growth into neoclassical growth theory and to use the Equivalence Principle (cf. Becker and Boyd (1997)) to argue that it is quite likely that the observed market rate of interest is too high from a public interest point of view. The idea is simple. Suppose that “natural capital”, i.e. the services of the environment in production are complementary with capital. To put it another way, suppose an increase in these services increases the marginal physical product of capital. Since the observed interest rate is the marginal physical product of capital, the observed interest rate increases with an increase in environmental services. Now suppose, as is plausible, that the services of the environment are being over used from the public interest point of view. Then it follows that if one uses the observed rate of interest as a dis-
count rate (after adjusting for taxes and other distortions) for Benefit/Cost analysis, then one is discounting too heavily. Since many conservation projects forgo development benefits today in return for rewards tomorrow, the discounting effect discriminates against environmental projects. However, Hochman, and Hochman (1980), have shown that the effect can go the other way if there is a shortage (measured relative to the public interest) of complementary public capital such as roads and bridges. Hence, one must “net out” these possibly canceling effects from a paucity of public infra structural capital (even if one has strong evidence that environmental services are being over used from the public interest point of view) to get the net bias in the observed market rate of interest.

The Brock (1977) paper sets out a framework for optimal growth with pollution cost that can be used as a building block to study the costs of irreversibility when uncertainty is added as in Brock and Mirman (1972). A recent Thesis, Pizer (1996), builds on Brock and Mirman (1972) to show that the presence of irreversibility in the context of global climate policy leads to a precautionary principle. Current research in Carpenter, Ludwig, and Brock (1998) is exploring infinite horizon stochastic optimal management models that may have multiple basins of attraction when the shocks are set to zero in order to explore the impact of potential hysteretic effects (e.g. getting trapped in a low welfare basin of attraction which is difficult to get out of) on optimal management policy.

Brock and Evans (1985) as well as the book on the economics of small business, Brock and Evans (1986), attack the rather practical policy problem of designing regulations in actual practice. We study the problem of administering pollution taxes on individual emitters when there are administrative costs both at the individual complier level and the bureau level. The paper and book extend received theories of the equilibrium size distribution of business firms to develop an argument for progressive taxes where the tax per unit emission is zero up to a certain size class, rises progressively through a range of size classes to reach marginal social cost for the largest size classes. The paper argued that this progressivity in regulatory burden was in the overall social interest because fixed administrative compliance costs as the individual firm level created an artificial scale economy which distorted the equilibrium size distribution of business firms away from the efficient size distribution. This distortion created a tradeoff between two different types of distortions which regulatory tiering optimizes.

It may be that much of the hostility to government regulation in the
U.S. is due to cumbersome, inflexible, “one size fits all,” and “command and control” type regulation. In our book on small business we reviewed empirical work that documents an approximately lognormal distribution of business sizes, however size is measured. This suggests that problem causers may be approximately log normally distributed and that regulatory efforts should be targeted towards the small number of big polluters found in the right hand tail of the distribution.

It is plausible to believe that part of the hostility of the average business person towards regulation may be due to anxiety on the part of the regulatees in forecasting and understanding the behavior of the regulators. This unpredictability of the impact of a regulation (and the unpredictability of the behavior of the regulators who carry it out) can generate a “risk cost” which can act like a fixed cost upon each business person.

Regulatory tiering theory gives a systematic common sense logical structure which one can use to design regulations that essentially go after the real problem causers and essentially leave minor problem causers alone. Obviously this method generates a lot less political backlash in a one person one vote political system.

Regulatory tiering theory recommends something like a progressive reduction of burden from the heaviest burden being placed on the largest to the least burden (maybe no burden at all) placed on the smallest. Such a policy is much easier to sell to legislatures because there are more votes embodied in the small operations, their families, friends, and relatives.

Brock (1980) represents an extension of the basic Groves and Ledyard (1977) revelation mechanism to various settings not covered by Groves and Ledyard. The paper also contains an easy constructive method of generating large classes of such preference revelation mechanisms.

Section D: Nonlinearity in Economics

This Section contains six papers. They are: 1. Brock and Dechert (1988); 2. Brock and Baek (1991); 3. Baek and Brock (1992); 4. Brock, Dechert, Scheinkman, and LeBaron (1996); 5. Brock (1997); and 6. Brock and Hommes (1997). The first four papers represent more recent work on designing statistical tests for the presence of nonlinear data generating mechanisms in economic data as well as understanding economic forces that might lead to patterns in data that are not well approximated by transforms of linear stochastic processes such as Auto Regressive Integrated Moving Average (AR-
IMA) processes driven by Generalized AutoRegressive Conditionally Heteroskedastic (GARCH) errors. This represents part of my continuing obsession with the study of patterns in economic data.

Some of the work in this section has generated interest in other sciences besides economics. See, for example, Grenfell, May, and Tong (1994), Weigend and Gershenfeld (1994), and Green and Savit (1991). Books that are targeted to the larger public that discuss some of the work above include Casti (1990), Peters (1994), and Vaga (1994).

The first work I presented in this area was in 1985 at a conference in Paris which was organized by J. M. Grandmont. This activity resulted in Brock (1986) where I reported on efforts to test for the possibility of a deterministic chaotic process lurking in the background of stochastic appearing patterns in economic data.

The evidence for chaos turned out to be weak, but different patterns in economic and financial data have been detected with some of the statistical methods that have been developed since 1985. Some of this work is contained in the first five papers. The sixth paper not only reports on recent modeling efforts in an attempt to understand some of the patterns that have been found, but also gives some of the history behind this type of work. The fourth paper an extension of the original 1987 BDS paper which ultimately ended up being used as a specification test in time series econometrics (See, for example, the article by Bollerslev, Engle, and Nelson (1994)).

Departures from conventional models in finance detected by methods like those above caused Brock, Lakonishok, and LeBaron (1992) to develop their bootstrap-based specification test. This method bootstraps the null model’s distribution of profit statistics gleaned from a trading strategy. If this method rejects the null model it gives economically interpretable information about where the model is going wrong. However, this method, unlike the BDS method, is more computer and time intensive.

Thus a useful practice in an applied field like finance is to first use something like the BDS test to suggest existence of departures from the null model and then use a method where the statistical quantities are of direct interest to the particular field (for example, trading strategies in financial applications) like a bootstrap-based specification test with trading rules. The methodology behind bootstrap-based tests build on the idea that the statistics bootstrapped should reflect the goals of the actors whose equilibrational activities are supposedly shaping the patterns which are supposed to be captured by the null model under scrutiny.
The last two papers in this section study a class of models where conventional rational expectations are nested within a general evolutionary setting where different predictors including rational expectations, compete on the basis of a performance measure. The objective is to develop econometrically tractable structures which can be taken to data and which build on existing work on rational expectations models, some of which was reviewed above. In these models rational expectations can be set up as a null hypothesis. The variation in size of rejections against different alternative boundedly rational expectations can be varied in an attempt to get some understanding of the sources of patterns in the data. See Baak (1998) and Chavas (1995) for ongoing work in this direction.

When one specifies a particular form of bounded rationality as the alternative, the rejection of the null may be caused by incorrect specification of the form of bounded rationality that is present (as well as incorrect specification of the null hypothesis of rationality). Brock (1997) sketches a theory of a flexibly parameterized space of boundedly rational predictors where probability mass accumulates on each as a function of the evolution of performance over time. In equilibrium, expressions that resemble estimators of sample moments appear. The analogy principle in econometrics suggests development of a limit theory and that theory is sketched in the paper. Development of this theory constitutes an ongoing research program, some of which appears in working papers with Hommes listed in the attached Curriculum Vitae of Brock.
References


(1986): *The Economics of Small Businesses: Their Role and Regulation in the U.S. Economy*. Holmes and Meier, New York.


